# Testing Large Time-Variations in Factor Models

Wen Xu*
Department of Economics & Oxford-Man Institute
University of Oxford

(IN PROGRESS, COMMENTS WELCOME)

**Abstract**

Principal component estimates of the factors are inconsistent when there are large temporal instabilities in factor loadings in approximate factor models (Bates et al., 2013). In this paper we test whether there has been persistent time-variation in factor loadings since a potential changepoint, which can be near the end of the sample. Both $I(0)$ and $I(1)$ factors are taken into account. We show the asymptotics in which the sizes of subsamples before and after the changepoint are both infinite. Monte Carlo studies have good results in finite samples. We apply the tests to a panel of UK macroeconomic and financial variables to check whether there was persistent instability during and after the recent financial crisis.

## 1 Introduction

Factor models have been important in the analysis of high dimensional dataset in macroeconomics and finance (Bai and Ng, 2008b; Stock and Watson, 2011), by extracting information from hundreds of economic variables and leading to the improvement of forecast or nowcast accuracy (see e.g. Stock and Watson (1999) and Giannone et al. (2008)). In practice such models are systemically used and perform well in the institutions like the Fed and the European Central Bank, but fail to precisely nowcast the GDP growth of United Kingdom during and after the recent financial crisis. Temporal instability in factor loadings is a potential reason for poor forecasting performance (Stock and Watson, 2009). The inference on factors are still valid provided that the instability is not so strong, as shown by Bates et al. (2013) who studied the type of instabilities in factor loadings including single large breaks and time-varying loadings among others, and its magnitude under which the principal component (PC) estimator of the factors is consistent and the number of factors can be consistently estimated using the information criterion (IC) in Bai and Ng (2002). If time variation is stronger, however, the inference will be misleading and the forecast will be affected. Testing time-variation in

*This version is preliminary. Email: wen.xu@economics.ox.ac.uk. We thank Bent Nielsen and Kevin Sheppard for helpful advices.

factor loadings is therefore necessary before every serious application of factor models. If there is strong temporal instability, a factor model with time-varying loadings (Del Negro and Otrok, 2008) might be a better candidate used for forecasting.

The literature on testing loading instability focuses on breaks, including Breitung and Eickmeier (2011), Chen et al. (2014), Han and Inoue (forthcoming), Yamamoto and Tanaka (2013), Corradi and Swanson (2014) and Cheng et al. (2014). A factor model with breaks in loadings can be equivalently represented by another factor model with constant loadings but a larger number of factors. Consequently, if the number of factors is not imposed ex-ante in the factor-augmented regression but estimated by the IC of Bai and Ng (2002), breaks in loadings have no influence on the parameter estimates in the regression. Factor models with time-varying loadings have no equivalent representation of stable models and the number of factors can not in general be estimated consistently using the existing methods in the literature (Takongmo and Stevanovic, 2014). Time-varying loadings, in this sense, are more serious an issue that potentially affects the factor-augmented regression and need to be tested. In addition, a model with multiple breaks is a special case of a model with time-varying parameters. Testing parameter stability against the alternative of martingales is addressed in a general maximum likelihood framework in Nyblom (1989) and extended to the models with integrated regressors in Hansen (1992) and panel data models in Yamazaki and Kurozumi (2014). This approximate Lagrange multiplier (LM) test does not require the knowledge of the breakpoint and is locally most powerful.

In this paper, we test large time-variation in factor loadings in approximate factor models in presence of both cross sectional dependence and heteroskedasticity. The magnitude of instability we intend to test is large enough to invalidate the effectiveness of PC estimates. Different from the literature of time-varying coefficients, the time-variation in the alternative hypothesis starts at a potential changepoint, which can be close to the end of the sample. Therefore, we can use the stable subsample to consistently estimate the number of factors as well as the error covariance matrix, in order to avoid the issue of non-monotonic power. Our test takes into account of both $I(0)$ and $I(1)$ factors with different distribution results. The major concern is the accumulation of factor estimate errors over time which might weakens the success of the tests in conventional models. Our results show that despite the strong restriction between $N$ and $T$ required for test statistics to follow the limit null distribution, it becomes much looser in finite sample simulation studies. We apply our test to a UK dataset to check whether there was considerable time-variation during and after the recent financial crisis. Section 2 provides the details on the models and assumptions. The asymptotics in which the subsample before and after the changepoint are both infinitely long are discussed in Section 2 with $I(0)$ factor and Section 3 with $I(1)$ factors. Section 4 presents the Monte Carlo results. We apply our test to the UK dataset in Section 5. Section 6 concludes the paper.

## 2 Models and asymptotics for stationary factors

We study the approximate factor model with time-varying factor loadings with the following data generation process[1]

$$X_{it} = \lambda_{it}' F_t + u_{it}$$

where

$$\lambda_{it} = \begin{cases} \lambda_{i0} & t \leq T_a \\ \lambda_{i,t-1} + e_{it} & t > T_a \end{cases}$$

The changepoint $T_a$ can be somewhere in the middle of the sample or near the end. In the former case, we will use asymptotics in which $T_a$ and $\tilde{T} = T - T_a$ are both infinite. In the latter case, we will assume that $\tilde{T}$ is fixed. The model has the following matrix form

$$\mathrm{vec}(X') = \mathrm{vec}(\Lambda_0 F') + \underbrace{D_F (L \otimes I_{kN}) e}_{v} + u$$

where $X = (X_1, \ldots, X_T)'$; $F = (F_1, \ldots, F_T)'$; $\Lambda_0 = (\lambda_{10}, \ldots, \lambda_{N0})'$; $D_F = \mathrm{diag}(I_N \otimes F_1', \ldots, I_N \otimes F_T')$; $e = (e_1', \ldots, e_T')'$ where $e_t = (e_{1t}', \ldots, e_{Nt}')'$; $u = (u_1', \ldots, u_T')'$; $L$ is a $T \times T$ lower triangular matrix with all non-zero elements equal to 1; $v_t = (I_N \otimes F_t') \sum_{s=1}^{t} e_s$.

Throughout the paper, $\| A \| = \sqrt{tr(A'A)}$ is the Frobenius norm; $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the minimum and maximum eigenvalues of $A$. We make the following assumptions on factors, loadings and idiosyncratics for asymptotics of the case where $\tilde{T} \to \infty$ and $F_t$ is $I(0)$.

**Assumptions**

1. (Factors) $F_t$ is stationary with $\mathrm{E}(F_t F_t') = \Sigma_F$ and $\mathrm{E} \| F_t \|^4 < \infty$.
2. (Initial loadings)
   (a) $\| \lambda_{i0} \| \leq \overline{\lambda} < \infty$.
   (b) $||\Lambda_0' \Lambda_0 / N - \Sigma_\Lambda|| \to 0$ as $N \to \infty$ for some positive definite $\Sigma_\Lambda$.
3. (Loading dynamics)
   (a) $\mathrm{E}(e_{it}) = 0$ and $\mathrm{E}(e_{it} e_{it}') = h_{NT} \Sigma_e$ which is known and positive definite.
   (b) $e_{it}$ is i.i.d. across both $i$ and $t$ .
   (c) $e$ is independent of both $F$ and $u$ .
4. (Idiosyncratics)
   (a) $u_t$ is i.i.d. with $\mathrm{E}(u_t) = 0$, $\mathrm{E}(u_t u_t') = V$ and $\mathrm{E}(u_{it}^8) < \infty$.

[1]It is allowed to have small time-variation in loadings when $t \leq T_a$ such that the factors and their number can be consistently estimated by PC and Bai and Ng (2002)'s IC, respectively.

(b) $\max_i \sum_{k=1}^{N} |V_{ki}| \leq M$ for all $N$ .

(c) $\mathrm{E}|N^{-1/2}\sum_{i=1}^{N}[u_{is}u_{it} - \mathrm{E}(u_{is}u_{it})]|^4 \leq M$ for every $s$ and $t$.

(d) For every $t$, $\mathrm{E}||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N} F_s[u_{is}u_{it} - \mathrm{E}(u_{is}u_{it})]||^2 \leq M$.

(e) $\mathrm{E}||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N} F_s u_{is}\lambda_i'||^2 \leq M$.

(f) For every $t$, $\mathrm{E}||\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\lambda_i u_{it}||^4 \leq M$ .

(g) $u$ and $F$ are independent.

(h) Let $V_{i.}^{-1}$ denote the $i$th row of $V^{-1}$ and $V_{ij}^{-1}$ is the $ij$th element of $V^{-1}$.

   i. The number of non-zero elements in $V_{i.}^{-1}$ is $r_{i,N}$ which is finite when $N \to \infty$.

   ii. The number of non-zero elements in $\hat{V}_{i.}^{-1}$ is $\hat{r}_{i,N}$ which is finite almost surely when $N \to \infty$.

   iii. $\hat{V}_{ij}^{-1} \xrightarrow{p} V_{ij}^{-1}$ for each $i, j, N$.

**Comments on assumptions:**

**4(b)-(f)** are borrowed from Bai (2003) in order to consistently estimate the factors and the number of factors.

**4(hi)**: Examples include cross-sectional heteroskedasticity with no dependence, cross-sectional heteroskedasticity with block-diagonal correlation structure of fixed block size (Choi, 2012), AR(1) structure, among others. Block-diagonal structure is motivated by the fact that in macroeconomic or financial applications when the idiosyncratic components represent industry-specific shocks, they are almost uncorrelated among the variables across different industries.

**4(hii-hiii)** There are two existing approaches to consistently estimate the idiosyncratic covariance matrix in approximate factor models. One is simply $\hat{V}_{simple} = \frac{1}{T}\sum_{t=1}^{T}\hat{u}_t\hat{u}_t'$ provided we have the prior knowledge[2] on the structure of the covariance matrix (e.g. block diagonal)(Choi, 2012). The other approach is the principal orthogonal complement thresholding (POET) (Fan et al., 2013; Bai and Liao, 2013) in which $\hat{V}_{poet} = s(\frac{1}{T}\sum_{t=1}^{T}\hat{u}_t\hat{u}_t')$ where $s$ is a general thresholding function. Their method requires further primitive assumptions on the idiosyncratics[3]. They show if $\frac{1}{\sqrt{N}} + \sqrt{\frac{logN}{T_a}} \to 0$, then $||\hat{V}_{poet}^{-1} - V^{-1}||_{spec} = o_p(1)$[4] which implies $\hat{V}_{ij,poet}^{-1} \xrightarrow{p} V_{ij}^{-1}$.

We intend to test whether there has been time-variation since a changepoint in the sample period, especially considerable variations that affect the consistent estimation of the factors and their number. Bates et al. (2013) showed in general we need $\sqrt{h_{NT}} = O(1/\min(T,(NT)^{1/2}))$ for consistency of Bai and Ng (2002)'s estimator of the factor number and $\sqrt{h_{NT}} = o(T^{-1/2})$ for mean square consistency of the PC estimator of factors. Thus, the desired test would have non-trivial power against larger variations, such as $h_{NT} = a/T$ with $a$ fixed. The null and alternative hypothesis are

[2] If we don't have the prior knowledge, $\hat{V}_{simple}$ is nonsingular.

[3] 1) There is a constant $c_1$ such that $\lambda_{min}(V) > c_1$ and $\min_{i,j} \mathrm{var}(u_{it}u_{jt}) > c_1$. 2) There are constants $r_1, r_2, b_1, b_2 > 0$ such that $P(|u_{it}| > s) \leq \exp(-(\frac{s}{b_1})^{r_1})$ and $P(|f_{jt}| > s) \leq \exp(-(\frac{s}{b_2})^{r_2})$ for any $s > 0$.

[4] $||A||_{spec} = \lambda_{\max}^{1/2}(A'A)$

$$
\begin{aligned}
H_{0A}: \quad h_{NT} &= 0 \\
H_{1A}: \quad h_{NT} &> 0
\end{aligned}
$$

We test against the alternative hypothesis for the factor loading of a particular variable, similar to Breitung and Eickmeier (2011). That is, we assume there is time variation only in the loading $\lambda_i$ and no time variation in loadings $j \neq i$. We will investigate the likelihood during the sample period $T_a+1:T$. For ease of notations, $X$ and $F$ represent $X_{T_a+1:T}$ and $F_{T_a+1:T}$ respectively, and $L$ is a $\tilde{T}\times\tilde{T}$ lower triangular matrix with all non-zero elements equal to 1 in this section. Under normality, the conditional log-likelihood is

$$
l_{\text{vec}(X')|F}(\Lambda_0, h_{NT}, V, \Sigma_e) = -\frac{\tilde{T}}{2}\log(2\pi) - \frac{1}{2}\log|\Sigma(h_{NT})|
$$
$$
-\frac{1}{2}[\text{vec}(X') - \text{vec}(\Lambda_0 F')]'\Sigma^{-1}(h_{NT})[\text{vec}(X') - \text{vec}(\Lambda_0 F')]
$$

where $\Sigma(h_{NT}) = I_{\tilde{T}} \otimes V + h_{NT} D_F (L \otimes I_{kN})(I_{\tilde{T}} \otimes \tilde{\Sigma}_e)(L' \otimes I_{kN}) D_F'$ where $\tilde{\Sigma}_e$ is a $kN \times kN$ sparse matrix with the only non-zero $k\times k$ block from the $ik+1$th element to the $(i+1)k$th element on the diagonal being equal to $\Sigma_e$. The CMLE estimator of $\Lambda_0$ under the null hypothesis $h_{NT}=0$ is

$$
\hat{\Lambda}_0 = X'F(F'F)^{-1}
$$

and the estimated residual $\dot{u}+\dot{v} = \text{vec}(X') - \text{vec}(\hat{\Lambda}_0 F') = (M_F \otimes I_N)\text{vec}(X') = (M_F \otimes I_N)(u+v)$ where $M_F = I_{\tilde{T}} - F(F'F)^{-1}F'$.

Nyblom (1989) recommended the approximate LM statistic to test the null hypothesis. In our settings, the self-normalized statistic for individual $i$ can be expressed as

$$
\text{LM}_i = \frac{1}{\tilde{T}^2 V_{ii}^{-1}} \sum_{\tau=1}^{\tilde{T}} \sum_{t=T_a+\tau}^{T} \frac{\partial f(X_t|F_t)'}{\partial \text{vec}(\Lambda')} G \sum_{t=T_a+\tau}^{T} \frac{\partial f(X_t|F_t)}{\partial \text{vec}(\Lambda')}
$$

where $f(X_t|F_t)$ is the Gaussian conditional density; $G = C_i \otimes \Sigma_e = (C_i \otimes \Sigma_{ec})(C_i \otimes \Sigma_{ec}')$ where $C_i$ is a $N\times N$ matrix with the $i$th diagonal being 1 and the others being 0 and $\Sigma_e = \Sigma_{ec}\Sigma_{ec}'$ is the Cholesky decomposition. As

$$
\begin{aligned}
\frac{\partial f(X_t|F_t)}{\partial \text{vec}(\Lambda')} &= \text{vec}(F_t X_t' V^{-1}) - \text{vec}(F_t F_t' \hat{\Lambda}' V^{-1}) \\
&= \text{vec}(F_t X_t' V^{-1}) - \text{vec}(F_t F_t' (F'F)^{-1} F' X V^{-1})
\end{aligned}
$$

and

$$\begin{aligned}
\sum_{t=T_a+\tau}^{T}(C_i\otimes\Sigma_{ec}')\frac{\partial f(X_t|F_t)}{\partial \mathrm{vec}(\Lambda')} &= \sum_{t=T_a+\tau}^{T}(C_i\otimes\Sigma_{ec}')[\mathrm{vec}(F_tX_t'V^{-1})-\mathrm{vec}(F_tF_t'(F'F)^{-1}F'XV^{-1})] \\
&= \sum_{t=T_a+\tau}^{T}(C_i\otimes\Sigma_{ec}')[(V^{-1}X_t)\otimes F_t-(V^{-1}X'F(F'F)^{-1}F_t)\otimes F_t] \\
&= \sum_{t=T_a+\tau}^{T}[(V_{i.}^{-1}X_t)\otimes(\Sigma_{ec}'F_t)-(V_{i.}^{-1}X'F(F'F)^{-1}F_t)\otimes(\Sigma_{ec}'F_t)] \\
&= \Sigma_{ec}'[\sum_{t=T_a+\tau}^{T}F_tV_{i.}^{-1}X_t-\sum_{t=T_a+\tau}^{T}F_tF_t'(\sum_{t=T_a+1}^{T}F_tF_t')^{-1}\sum_{t=T_a+1}^{T}F_tV_{i.}^{-1}X_t]
\end{aligned}$$

In sum,

$$\mathrm{LM}_i=\frac{1}{\tilde{T}^2V_{ii}^{-1}}\sum_{\tau=1}^{\tilde{T}}s_{i\tau}'\Sigma_e s_{i\tau}$$

where

$$\begin{aligned}
s_{i,[\tilde{T}r]} &= \sum_{t=T_a+1}^{[\tilde{T}r]+T_a}F_tV_{i.}^{-1}(u_t+v_t)-\sum_{t=T_a+1}^{[\tilde{T}r]+T_a}F_tF_t'(\sum_{t=T_a+1}^{T}F_tF_t')^{-1}\sum_{t=T_a+1}^{T}F_tV_{i.}^{-1}(u_t+v_t) \\
&= \sum_{t=T_a+1}^{[\tilde{T}r]+T_a}F_tV_{i.}^{-1}X_t-\sum_{t=T_a+1}^{[\tilde{T}r]+T_a}F_tF_t'(\sum_{t=T_a+1}^{T}F_tF_t')^{-1}\sum_{t=T_a+1}^{T}F_tV_{i.}^{-1}X_t
\end{aligned}$$

Under the null hypothesis and assumptions 1 and 4, as $\tilde{T}\to\infty$, $\frac{1}{\sqrt{\tilde{T}}}\sum_{t=T_a+1}^{[\tilde{T}r]+T_a}F_tV_{i.}^{-1}u_t\xrightarrow{\mathrm{D}}\sqrt{V_{ii}^{-1}}\Sigma_F^{1/2}W_1(r)$ where $W_1$ is a $k$-dimensional standard Brownian motion and therefore

$$\frac{1}{\sqrt{\tilde{T}V_{ii}^{-1}}}s_{i,[\tilde{T}r]}\xrightarrow{\mathrm{D}}\Sigma_F^{1/2}(W_1(r)-rW_1(1))$$

$$\mathrm{LM}_i\xrightarrow{\mathrm{D}}\int_0^1(W_1(r)-rW_1(1))'\Sigma_F^{1/2}\Sigma_e\Sigma_F^{1/2}(W_1(r)-rW_1(1))dr$$

Let $\tilde{F}_t=H'F_t$ where $H=(\Lambda_0'\Lambda_0/N)(F'\hat{F}/T)V_{NT}^{-1}$; $\hat{F}$ is $\sqrt{T}$ times the eigenvectors corresponding to the $k$ largest eigenvalues of the matrix $XX'$ and $V_{NT}$ is the $k\times k$ diagonal matrix of the first $k$ largest eigenvalues of $XX'/(TN)$, as in Bai (2003). In practice, we can consistently estimate the rotated factors $\tilde{F}_t$ rather than $F_t$ . Rewriting $s_{i,[\tilde{T}r]}$ and $\mathrm{LM}_i$ by replacing $F_t$ with $\tilde{F}_t$ yields $\tilde{s}_{i,[\tilde{T}r]}$ and $\widetilde{\mathrm{LM}}_i$ such that

$$\tilde{s}_{i,[\tilde{T}r]}=H's_{i,[\tilde{T}r]}$$

Thus, as $N,T\to\infty$,

$$\frac{1}{\sqrt{\tilde{T}V_{ii}^{-1}}}\tilde{s}_{i,[\tilde{T}r]}\xrightarrow{\mathrm{D}}\tilde{H}'\Sigma_F^{1/2}(W_1(r)-rW_1(1))$$

$$\widetilde{\mathrm{LM}}_i \xrightarrow{\mathrm{D}} \int_0^1 (W_1(r) - rW_1(1))' \Sigma_F^{1/2} \tilde{H} \Sigma_e \tilde{H}' \Sigma_F^{1/2} (W_1(r) - rW_1(1)) dr$$

where $\tilde{H} = \underset{N,T \to \infty}{\text{plim}} H$.

The limit distribution of $\widetilde{\mathrm{LM}}_i$ can be free from nuisance parameters, provided $\Sigma_e$ is replaced with $\tilde{\Sigma}_F^{-1}$ where $\tilde{\Sigma}_F = \tilde{H}' \Sigma_F \tilde{H} = \underset{\mathrm{N,T} \to \infty}{\text{plim}} (\frac{1}{T} \sum_{t=1}^{T} \tilde{F}_t \tilde{F}_t')$. Therefore,

$$\widetilde{\mathrm{LM}}_i = \frac{1}{\tilde{T}^2 V_{ii}^{-1}} \sum_{\tau=1}^{\tilde{T}} \tilde{s}_{i\tau} \tilde{\Sigma}_F^{-1} \tilde{s}_{i\tau}$$

$$\widetilde{\mathrm{LM}}_i \xrightarrow{\mathrm{D}} \int_0^1 (W_1(r) - rW_1(1))' (W_1(r) - rW_1(1)) dr$$

which is the generalized Von Mises distribution with $k$ degrees of freedom.

In practice, we need to estimate the number of factors $k$, the factors, $\tilde{\Sigma}_F$ and the idiosyncratic covariance matrix $V$. We use the subsample from $t = 1$ to $T_a$ to consistently estimate $k$, by Bai and Ng (2002)'s IC, while the factors are estimated using PC for the full sample.

$$\widehat{\mathrm{LM}}_i = \frac{1}{\tilde{T}^2 \hat{V}_{ii}^{-1}} \sum_{\tau=1}^{\tilde{T}} \hat{s}_{i\tau} \hat{\Sigma}_F^{-1} \hat{s}_{i\tau}$$

where

$$\hat{s}_{i,[\tilde{T}r]} = \sum_{t=T_a+1}^{[\tilde{T}r]+T_a} \hat{F}_t \hat{V}_{i.}^{-1} X_t - \sum_{t=T_a+1}^{[\tilde{T}r]+T_a} \hat{F}_t \hat{F}_t' (\sum_{t=T_a+1}^{T} \hat{F}_t \hat{F}_t')^{-1} \sum_{t=T_a+1}^{T} \hat{F}_t \hat{V}_{i.}^{-1} X_t$$

and $\hat{\Sigma}_F = \frac{1}{T_a} \sum_{t=1}^{T_a} \hat{F}_t \hat{F}_t'$ can effectively avoid the issue of non-monotonic power.

**Theorem 1**: Under the null hypothesis, if $\frac{\tilde{T}}{\sqrt{\min(N,T^2)}} = o(1)$ , then $\widehat{\mathrm{LM}}_i = \widetilde{\mathrm{LM}}_i + o_p(1)$ as $N, T, \tilde{T} \to \infty$ . Thus, $\widehat{\mathrm{LM}}_i \xrightarrow{\mathrm{D}} \int_0^1 (W_1(r) - rW_1(1))' (W_1(r) - rW_1(1)) dr$.

**Comments on Theorem 1:** $\frac{\tilde{T}}{\sqrt{\min(N,T^2)}} = o(1)$ seems strict and requires a small $\tilde{T}$. In simulations, however, the test with a moderate $\tilde{T}$ can still have good size and power.

# 3 Models and asymptotics for $I(1)$ factors

In this section, factors are also $I(1)$ in the period from $T_a$ to $T$ with no cointegration among them. Specifically, $F_t = F_{t-1} + \varepsilon_t$ when $t > T_a$ where $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$. Assumptions 4(d)-(e) don't hold any longer. Now the statistic is defined as

$$\mathrm{LM}_i = \frac{1}{\tilde{T}^3 \hat{V}_{ii}^{-1}} \sum_{\tau=1}^{\tilde{T}} s_{i\tau}' \Sigma_e s_{i\tau}$$

| | 1% | 5% | 10% |
|---|---|---|---|
| k=1 | 0.493 | 0.222 | 0.137 |
| k=2 | 0.581 | 0.294 | 0.208 |
| k=3 | 0.614 | 0.323 | 0.237 |
| k=4 | 0.626 | 0.342 | 0.252 |

Table 1: Critical values of the distribution $\int_0^1 \tilde{W}(r)'\tilde{W}(r)dr$ by simulation

Under Assumptions 1 and 4, $\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{[\tilde{T}r]+T_a} F_tF_t' \xrightarrow{D} \Sigma_\varepsilon^{1/2}\int_0^r W_1(\tilde{r})W_1'(\tilde{r})d\tilde{r}\Sigma_\varepsilon^{1/2}$ and $\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{[\tilde{T}r]+T_a} F_tV_{i.}^{-1}u_t \xrightarrow{D} \int_0^r \sqrt{V_{ii}^{-1}}\Sigma_\varepsilon^{1/2}W_1(\tilde{r})dW_2(\tilde{r})$ where $W_2$ is a standard Brownian motion and independent of $W_1$, so

$$\frac{1}{\tilde{T}\sqrt{V_{ii}^{-1}}}s_{i,[\tilde{T}r]} \xrightarrow{D} \Sigma_\varepsilon^{1/2}\tilde{W}(r)$$

where $\tilde{W}(r) = \int_0^r W_1(\tilde{r})dW_2(\tilde{r}) - \int_0^r W_1(\tilde{r})W_1'(\tilde{r})d\tilde{r}(\int_0^1 W_1(\tilde{r})W_1'(\tilde{r})d\tilde{r})^{-1}\int_0^1 W_1(\tilde{r})dW_2(\tilde{r})$. Consequently,

$$\text{LM}_i \xrightarrow{D} \int_0^1 \tilde{W}(r)'\Sigma_\varepsilon^{1/2}\Sigma_e\Sigma_\varepsilon^{1/2}\tilde{W}(r)dr$$

As for stationary factors, one can rotate the factor $\tilde{F}_t = H'F_t$ where $H = (\Lambda_0'\Lambda_0/N)(F'\hat{F}/T^2)V_{NT}^{-1}$; $\hat{F}$ is $T$ times the eigenvectors corresponding to the $k$ largest eigenvalues of the matrix $XX'$ and $V_{NT}$ is the $k\times k$ diagonal matrix of the first $k$ largest eigenvalues of $XX'/(T^2N)$, as in Bai (2004). Similarly above in Section 2, we define $\widetilde{\text{LM}}_i = \frac{1}{\tilde{T}^3V_{ii}^{-1}}\sum_{\tau=1}^{\tilde{T}}\tilde{s}_{i\tau}\tilde{\Sigma}_\varepsilon^{-1}\tilde{s}_{i\tau}$ and

$$\widetilde{\text{LM}}_i \xrightarrow{D} \int_0^1 \tilde{W}(r)'\tilde{W}(r)dr$$

We estimate $\widetilde{\text{LM}}_i$ by $\widehat{\text{LM}}_i$ such that

$$\widehat{\text{LM}}_i = \frac{1}{\hat{V}_{ii}^{-1}\tilde{T}^2}\sum_{\tau=1}^{\tilde{T}}\hat{s}_{i\tau}\hat{\Sigma}_\varepsilon^{-1}\hat{s}_{i\tau}$$

where $\hat{\Sigma}_\varepsilon = \frac{1}{\tilde{T}-1}\sum_{t=T_a+1}^{T-1}(\hat{F}_{t+1}-\hat{F}_t)(\hat{F}_{t+1}-\hat{F}_t)'$.

**Theorem 2:** Under the null hypothesis, if $\frac{\tilde{T}^2}{\sqrt{\min(N\tau^2,\frac{T^3}{\tilde{T}})T}} = o(1)$ , then $\widehat{\text{LM}}_i = \widetilde{\text{LM}}_i + o_p(1)$ as $N,T,\tilde{T}\to\infty$ . Thus, $\widehat{\text{LM}}_i \xrightarrow{D} \int_0^1 \tilde{W}(r)'\tilde{W}(r)dr$.

# 4 Monte Carlo Results

In this section we present the simulation results of our test statistics in finite samples. We consider a two-factor model with $N = 50, 100, 150, 200$ and $T = 100, 150, 200, 250$, which are typical in empirical applications. $\{F_t\}$ is either i.i.d. $N(0,I_2)$ or random walk with $N(0,I_2)$ error terms. We use Bai and Ng (2002)'s IC to estimate the number of factors except for $N=50$, where we use Onatski (2010)'s empirical distribution of eigenvalues for better results. We study two kinds of idiosyncratic structure. One is the block diagonal covariance matrix with the size of each block being 5 (Choi, 2012), which satisfies Assumption 4(h)i. We generate each block by i.i.d. Wishart distribution $W_p(I_5,5)$. The other is the following banded structure from (Bai and Liao, 2013) which violates Assumption 4(h)i. We intend to investigate the effect of this assumption on the size and power in finite samples.

$$\begin{aligned} u_{1t} &= \tilde{u}_{1t} \\ u_{2t} &= \tilde{u}_{2t} + a_1\tilde{u}_{1t} \\ u_{3t} &= \tilde{u}_{3t} + a_2\tilde{u}_{2t} + b_1\tilde{u}_{1t} \\ u_{it} &= \tilde{u}_{it} + a_{i-1}\tilde{u}_{i-1,t} + b_{i-2}\tilde{u}_{i-2,t} + c_{i-3}\tilde{u}_{i-3,t},\ i \geq 4 \end{aligned}$$

where $a_i, b_i, c_i$ are i.i.d. $N(0,0.7^2)$ and $\{\tilde{u}_{it}\}$ are independent $N(0,1)$ across both time series and cross sections. The idiosyncratic covariance matrix is estimated by POET from Fan et al. (2013). Specifically,

$$\hat{V}_{ij} = \begin{cases} \frac{1}{T}\sum_{t=1}^{T}\hat{u}_{it}\hat{u}'_{jt} & \text{if } i=j \\ s(\frac{1}{T}\sum_{t=1}^{T}\hat{u}_{it}\hat{u}'_{jt}) & \text{if } i \neq j \end{cases}$$

where $\hat{u}_{it}$ is the residual from PC estimation and $s(x) = \text{sgn}(x)(|x| - \tau_{ij})$ is the soft thresholding rule with the threshold $\tau_{ij} = \sqrt{\hat{V}_{ii}\hat{V}_{jj}}(\sqrt{(logN)/T_a} + 1/\sqrt{N})$ from Bai and Liao (2013).

Loadings are generated by the random work, for $i = 1, \cdots, N$,

$$\lambda_{it} = \begin{cases} \lambda_{i0} & t \leq T_a \\ \lambda_{i,t-1} + e_{it} & t > T_a \end{cases}$$

where $\lambda_{i0}$ is i.i.d.$N(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, I_2)$; $\text{E}(e_{it}e'_{it}) = \frac{a}{T}\begin{bmatrix} 1 & \sigma_\lambda^2 \\ \sigma_\lambda^2 & 1 \end{bmatrix}$ such that $a^2 = 0,1,5,10,30,50$ and $\sigma_\lambda^2$ follows $Uniform(0,0.9)$; $T_a$ can be $\frac{T}{2}$, $T-10$ or $T-5$. In the latter two cases, $T - T_a$ is considered to become fixed. We apply Andrews (2003)'s subsampling-like method to compute the critical values.

Tables 2-5 show the simulation results. In general, the empirical sizes are close to the nominal size, but the rejection frequencies are decreasing in $N$ and $T$. When $N = 50$, it is sometimes oversized because the number of factors are more difficult to estimate in small samples. The power are also satisfied and it seems that the test is consistent in that the power grows to 1 as $N$ and $T$ increase. Violation of Assumption 4(h)i seems not affect the test results.

The tests designed for structural breaks, such as likelihood ratio, Wald and Lagrange multiplier statistics in Breitung and Eickmeier (2011), also have power for persistent variations. In Figure 4.1, We compare our LM statistic with them, which

| | Block Diagonal | | | | | |
|---|---|---|---|---|---|---|
| | I(0) Factors | | | I(1) Factors | | |
| | $T = 100$ | $T = 150$ | $T = 200$ | $T = 100$ | $T = 150$ | $T = 200$ |
| $N = 50$ | 0.049 | 0.041 | 0.032 | 0.092 | 0.066 | 0.057 |
| $N = 100$ | 0.053 | 0.037 | 0.034 | 0.037 | 0.023 | 0.020 |
| $N = 150$ | 0.050 | 0.033 | 0.025 | 0.032 | 0.029 | 0.024 |
| $N = 200$ | 0.040 | 0.031 | 0.024 | 0.036 | 0.023 | 0.024 |
| | Banded | | | | | |
| | I(0) Factors | | | I(1) Factors | | |
| | $T = 100$ | $T = 150$ | $T = 200$ | $T = 100$ | $T = 150$ | $T = 200$ |
| $N = 50$ | 0.082 | 0.050 | 0.046 | 0.042 | 0.033 | 0.034 |
| $N = 100$ | 0.060 | 0.044 | 0.038 | 0.043 | 0.037 | 0.030 |
| $N = 150$ | 0.058 | 0.042 | 0.035 | 0.046 | 0.034 | 0.031 |
| $N = 200$ | 0.055 | 0.038 | 0.029 | 0.043 | 0.033 | 0.030 |

Table 2: Empirical sizes for the models with $T_a = \frac{T}{2}$. The nominal size is 0.05 and 2500 replications are used.

| | Block Diagonal | | | | | |
|---|---|---|---|---|---|---|
| | I(0) Factors | | | I(1) Factors | | |
| | $T = 100$ | $T = 150$ | $T = 200$ | $T = 100$ | $T = 150$ | $T = 200$ |
| $a^2 = 1$ | 0.184 | 0.229 | 0.278 | 0.508 | 0.642 | 0.745 |
| $a^2 = 5$ | 0.832 | 0.907 | 0.938 | 0.940 | 0.972 | 0.980 |
| $a^2 = 10$ | 0.974 | 0.989 | 0.994 | 0.978 | 0.984 | 0.983 |
| $a^2 = 30$ | 0.997 | 0.998 | 0.998 | 0.891 | 0.923 | 0.936 |
| $a^2 = 50$ | 0.992 | 0.995 | 0.994 | 0.765 | 0.843 | 0.888 |
| | Banded | | | | | |
| | I(0) Factors | | | I(1) Factors | | |
| | $T = 100$ | $T = 150$ | $T = 200$ | $T = 100$ | $T = 150$ | $T = 200$ |
| $a^2 = 1$ | 0.269 | 0.343 | 0.402 | 0.612 | 0.746 | 0.824 |
| $a^2 = 5$ | 0.927 | 0.964 | 0.980 | 0.971 | 0.991 | 0.996 |
| $a^2 = 10$ | 0.995 | 0.998 | 1.000 | 0.993 | 0.996 | 0.996 |
| $a^2 = 30$ | 0.998 | 0.999 | 0.999 | 0.925 | 0.953 | 0.966 |
| $a^2 = 50$ | 0.997 | 0.997 | 0.997 | 0.835 | 0.897 | 0.924 |

Table 3: Power for the models with $T_a = \frac{T}{2}$.

| | Block Diagonal | | Banded | |
|---|---|---|---|---|
| | $\tilde{T} = 10$ | $\tilde{T} = 5$ | $\tilde{T} = 10$ | $\tilde{T} = 5$ |
| $N = 50$ | 0.069 | 0.052 | 0.068 | 0.057 |
| $N = 100$ | 0.070 | 0.053 | 0.070 | 0.059 |
| $N = 150$ | 0.068 | 0.056 | 0.071 | 0.065 |
| $N = 200$ | 0.067 | 0.060 | 0.071 | 0.062 |

Table 4: Empirical sizes for the models with $\tilde{T} = 5, 10$

| | Block Diagonal | | Banded | |
|---|---|---|---|---|
| | $\tilde{T}=10$ | $\tilde{T}=5$ | $\tilde{T}=10$ | $\tilde{T}=5$ |
| $a^2=5$ | 0.194 | 0.091 | 0.218 | 0.103 |
| $a^2=10$ | 0.398 | 0.158 | 0.458 | 0.181 |
| $a^2=30$ | 0.805 | 0.451 | 0.865 | 0.507 |
| $a^2=50$ | 0.920 | 0.619 | 0.957 | 0.680 |

Table 5: Power for the models with $\tilde{T}=5,10$

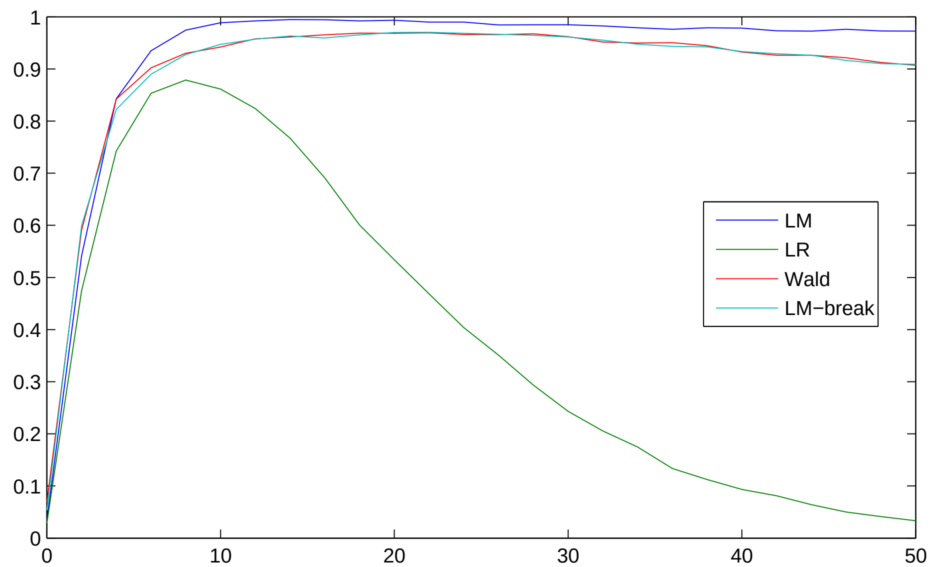


Figure 4.1: Power of test statistics for testing persistent variations, $T=150$, $N=100$, $I(0)$ factors, block diagonal idiosyncratics

assume there is a break at $T_a$. It is obvious that LR tests seriously suffer from the issue of non-monotonic power. The performances of Wald and LM for breaks are close to the LM for temporal instability, but are inferior to the latter when the variation is large. One should note that our LM test is specially designed to detect the temporal variations after the break point, while break tests can not tell what happens afterwards. As we said, persistent time variations are more harmful in the sense that the model can not be transformed into a stable one with increased number of factors.

# 5 Application

One application of factor models is nowcasting quarterly GDP growth using hundreds of predictors with different frequencies (Giannone et al. (2008) for Fed and Angelini et al. (2011) for ECB, for example). In practice, the performance is mixed and varies with the specific country to predict. It has good performance in the US and Euro Erea, but fails to work well in the UK during and after the recession. One can use various refinements, such as variable pre-selection prior to factor extraction and downweighting past information, but the improvement is limited after the recent financial crisis. It implies that structural instability is a major concern. One alternative is the factor model with time-varying loadings and stochastic volatility proposed in Del Negro and Otrok (2008). Therefore, it would be helpful to test whether there has been temporal instability since the crisis.

The raw dataset contains 152 series including 134 UK variables and 18 international variables from US and Germany listed in the Appendix. The series are either monthly or daily, and need to be transformed into a quarterly and stationary quantity. We use only the series for which there are less the 1/3 of missing data and adjust for ouliers and missing data to create a balanced dataset. Although the tradition is to use as many series as possible, one reason behind some unsatisfactory performance is perhaps the adverse influence of uninformative predictors for the target variable to forecast. Boivin and Ng (2006) showed that as few as 50 pre-selected predictors can yield better forecasts than 147 predictors. Recently several attempts have been made to improve PCA forecasts, including pre-selection of the predictors (Bai and Ng, 2008a) and boosting (Bai and Ng, 2009), among others. We pre-select 55 variables using the elasticity net, which is known as an effective tool to perform variable selection and shrinkage simultaneously (Bai and Ng, 2008a).

We test whether there was temporal instability after January 2007. Onatski (2010)'s criterion indicates that the number of factors is 11, 12 or 13 depending on the change-point, which is a little larger than that extracted from the US dataset. The relative rejection rate from 2007:01 to 2009:12 is shown in Figure 5.1. It can be seen that the time variation was most serious before 2009 but gradually faded away afterwards.

# 6 Conclusion

In the paper, we adapt Nyblom (1989)'s test statistic for factor models to check whether there is persistent time-variation in factor loadings, which might affect the consistency of factor estimates and therefore the forecasting performance of factor-augmented regressions. Although the tests designed for the alternatives of multiple breaks also have power for time-varying parameters, our statistic is more powerful to detect the persistence. Slightly strong restrictions are imposed on the relation between the cross-section dimension, the size of the whole sample and the subsample of temporal instability in order to achieve the desirable limit null distribution. In finite samples, however, the restrictions seem not to affect the empirical size and power. Also, we find the small effect of different idiosyncratic structure on the test performance. We apply the test statistic to the UK dataset and find that the time variation was most serious before 2009 but gradually faded away afterwards.


Figure 5.1: Relative rejection rates from 2007:01 to 2009:12

# Appendices

## .1 Proof of Theorem 1

Let $\delta_{NT} = \min(\sqrt{N}, \sqrt{T})$ and $J_{i,N} = \{j : V_{ij}^{-1} \neq 0 \, or \, \hat{V}_{ij}^{-1} \neq 0\}$. Under assumption 4(h), the number of elements in $J_{i,N}$ is finite almost surely when $N \to \infty$. Following Bai (2003),

$$\hat{F}_t - H'F_t = V_{NT}^{-1}(\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\zeta_{st} + \frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\eta_{st} + \frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\xi_{st} + \frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\gamma_{st})$$

where $\zeta_{st} = [u_s'u_t - \mathrm{E}(u_s'u_t)]/N$; $\eta_{st} = F_s'\Lambda'u_t/N$; $\xi_{st} = F_t'\Lambda'u_s/N$; $\gamma_{st} = \mathrm{E}(u_s'u_t)/N$. For convenience, we use the following notations

$$\hat{F}_t = B_{0t} + B_{1t} + B_{2t} + B_{3t} + B_{4t}$$

in which $B_{0t} = \tilde{F}_t$; $B_{1t} = V_{NT}^{-1}\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\zeta_{st}$, $B_{2t} = V_{NT}^{-1}\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\eta_{st}$, $B_{3t} = V_{NT}^{-1}\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\xi_{st}$, $B_{4t} = V_{NT}^{-1}\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\gamma_{st}$.

**Lemma 1.** *(Breitung and Eickmeier, 2011)* $\frac{1}{T}\sum_{t=1}^{T}\hat{F}_t\hat{F}_t' = \frac{1}{T}\sum_{t=1}^{T}\tilde{F}_t\tilde{F}_t' + O_P(\delta_{NT}^{-2}) = \tilde{\Sigma}_F + o_P(1)$. *Therefore,* $\hat{\Sigma}_F^{-1} = \tilde{\Sigma}_F^{-1} + o_P(1)$.

**Lemma 2.** $\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\zeta_{st}||^2 = O_P(\frac{\tilde{T}}{N}\delta_{NT}^{-2})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{1t}||^2 = O_P(\frac{\tilde{T}}{N}\delta_{NT}^{-2})$

**Lemma 3.** $\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\eta_{st}||^2 = O_P(\frac{\tilde{T}}{N})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{2t}||^2 = O_P(\frac{\tilde{T}}{N})$

**Lemma 4.** $\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\xi_{st}||^2 = O_P(\frac{\tilde{T}}{N}\delta_{NT}^{-2})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{3t}||^2 = O_P(\frac{\tilde{T}}{N}\delta_{NT}^{-2})$

**Lemma 5.** $\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\gamma_{st}||^2 = O_p(\frac{\tilde{T}}{T}\delta_{NT}^{-2})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{4t}||^2 = O_p(\frac{\tilde{T}}{T}\delta_{NT}^{-2})$

**Lemma 6.** $\frac{1}{T}\sum_{t=T_a+1}^{T}\hat{F}_t\hat{V}_{i.}^{-1}X_t = \frac{1}{T}\sum_{t=T_a+1}^{T}\tilde{F}_t V_{i.}^{-1}X_t + o_p(1)$ *when* $N,T\to\infty$.

**Lemma 7.** *(Bai, 2003)* $\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s - H'F_s||^2 = O_p(\delta_{NT}^{-2})$

**Lemma 8.** *(Bai, 2003)* $||H|| = O_p(1)$

**Proof of Lemma 2**

$$\begin{aligned}
\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\zeta_{st}||^2 &= \sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s - H'F_s + H'F_s)\zeta_{st}||^2 \\
&\leq \sum_{t=T_a+1}^{T}(||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\zeta_{st}|| + ||\frac{1}{T}\sum_{s=1}^{T}H'F_s\zeta_{st}||)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\zeta_{st}||^2 + \sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}H'F_s\zeta_{st}||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s - H'F_s||^2\frac{1}{NT}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}[N^{-\frac{1}{2}}\sum_{i=1}^{N}(u_{is}u_{it} - \mathrm{E}(u_{is}u_{it}))]^2 \\
&\quad + \frac{1}{NT}\sum_{t=T_a+1}^{T}||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N}F_s(u_{is}u_{it} - \mathrm{E}(u_{is}u_{it}))||^2||H'||^2) \\
&= O_p(\frac{\tilde{T}}{N}\delta_{NT}^{-2}) + O_p(\frac{\tilde{T}}{NT}) \\
&= O_p(\frac{\tilde{T}}{N}\delta_{NT}^{-2})
\end{aligned}$$

The third inequality holds by the Cauchy-Schwarz Inequality. The second identity uses Assumption 4(c), 4(d) and Lemma 7, 8. The lemma is proved following $||V_{NT}|| = O_p(1)$ from (Bai, 2003).

**Proof of Lemma 3**

$$
\begin{aligned}
\sum_{t=T_a+1}^{T} \|\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\eta_{st}\|^2 &= \sum_{t=T_a+1}^{T} \|\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s - H'F_s + H'F_s)\eta_{st}\|^2 \\
&\leq \sum_{t=T_a+1}^{T} (\|\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\eta_{st}\| + \|\frac{1}{T}\sum_{s=1}^{T}H'F_s\eta_{st}\|)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T} \|\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\eta_{st}\|^2 + \sum_{t=T_a+1}^{T} \|\frac{1}{T}\sum_{s=1}^{T}H'F_s\eta_{st}\|^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}\|\hat{F}_s - H'F_s\|^2 \frac{1}{N^2T}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}(F_s'\Lambda' u_t)^2 \\
&\quad + \sum_{t=T_a+1}^{T}\|\frac{1}{NT}\sum_{s=1}^{T}F_sF_s'\Lambda' u_t\|^2\|H'\|^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}\|\hat{F}_s - H'F_s\|^2 \frac{1}{NT}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}\|F_s'\|^2\|N^{-\frac{1}{2}}\Lambda' u_t\|^2 \\
&\quad + \|\frac{1}{T}\sum_{s=1}^{T}F_sF_s'\|^2\frac{1}{N}\sum_{t=T_a+1}^{T}\|N^{-\frac{1}{2}}\Lambda' u_t\|^2\|H'\|^2) \\
&= O_p(\frac{\tilde{T}}{N}\delta_{NT}^{-2}) + O_p(\frac{\tilde{T}}{N}) \\
&= O_p(\frac{\tilde{T}}{N})
\end{aligned}
$$

The second identity follows from Assumption 1, 4(f) and Lemma 7, 8.
**Proof of Lemma 4**

$$
\begin{aligned}
\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\xi_{st}||^2 &= \sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s-H'F_s+H'F_s)\xi_{st}||^2 \\
&\leq \sum_{t=T_a+1}^{T}(||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s-H'F_s)\xi_{st}||+||\frac{1}{T}\sum_{s=1}^{T}H'F_s\xi_{st}||)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s-H'F_s)\xi_{st}||^2+\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}H'F_s\xi_{st}||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s-H'F_s||^2\frac{1}{N^2T}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}(F_t'\Lambda'u_s)^2 \\
&\quad+\sum_{t=T_a+1}^{T}||\frac{1}{NT}\sum_{s=1}^{T}F_su_s'\Lambda F_t||^2||H'||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s-H'F_s||^2\frac{1}{NT}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}||F_t'||^2||N^{-\frac{1}{2}}\Lambda'u_s||^2 \\
&\quad+\frac{1}{NT}\sum_{t=T_a+1}^{T}||F_t||^2||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}F_su_s'\Lambda||^2||H'||^2) \\
&= O_p(\frac{\tilde{T}}{N}\delta_{NT}^{-2})+O_p(\frac{\tilde{T}}{NT}) \\
&= O_p(\frac{\tilde{T}}{N}\delta_{NT}^{-2})
\end{aligned}
$$

The second identity holds because of Assumption 1, 4(e), 4(f) and Lemma 7, 8.

**Proof of Lemma 5**

$$
\begin{aligned}
\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}\hat{F}_s\gamma_{st}||^2 &= \sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s-H'F_s+H'F_s)\gamma_{st}||^2 \\
&\leq \sum_{t=T_a+1}^{T}(||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s-H'F_s)\gamma_{st}||+||\frac{1}{T}\sum_{s=1}^{T}H'F_s\gamma_{st}||)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}(\hat{F}_s-H'F_s)\gamma_{st}||^2+\sum_{t=T_a+1}^{T}||\frac{1}{T}\sum_{s=1}^{T}H'F_s\gamma_{st}||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s-H'F_s||^2\frac{1}{N^2T}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}[\sum_{i=1}^{N}\mathrm{E}(u_{is}u_{it})]^2 \\
&\quad+\frac{1}{NT}\sum_{t=T_a+1}^{T}||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N}F_s\mathrm{E}(u_{is}u_{it})||^2||H'||^2) \\
&= O_p(\frac{\tilde{T}}{T}\delta_{NT}^{-2})+O_p(\frac{\tilde{T}}{T^2}) \\
&= O_p(\frac{\tilde{T}}{T}\delta_{NT}^{-2})
\end{aligned}
$$

The second identity is implied by Assumption 1, 4(a) and Lemma 7, 8.

**Proof of Lemma 6**

$$\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}\hat{F}_t\hat{V}_{i.}^{-1}X_t = \frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}\hat{V}_{ij}^{-1}X_{jt}$$
$$= \frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}(V_{ij}^{-1}+o_p(1))X_{jt}$$

Note that $\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}\tilde{F}_tV_{i.}^{-1}X_t = \frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}B_{0t}\sum_{j\in J_{i,N}}V_{ij}^{-1}X_{jt}$. The order of $\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}V_{ij}^{-1}X_{jt}$ is higher than that of $\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}o_p(1)X_{jt}$, since the number of elements in $J_{i,N}$ is finite almost surely when $N\to\infty$. Thus we focus on the $m$th ($m\neq 0$) bias term $\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}B_{mt}V_{ij}^{-1}X_{jt}$ which is is bounded by

$$||\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}B_{mt}V_{ij}^{-1}X_{jt}|| \leq \frac{1}{\tilde{T}}(\sum_{t=T_a+1}^{T}||B_{mt}||^2)^{\frac{1}{2}}(\sum_{t=T_a+1}^{T}(V_{ij}^{-1}X_{jt})^2)^{\frac{1}{2}}$$

By Lemmas 2-5, the dominant bias term is the 2th term or the 4th term depending on $\min(N,T^2)$, i.e. $||\frac{1}{\tilde{T}}\sum_{t=T_a+1}^{T}B_{mt}V_{ij}^{-1}X_{jt}|| = \frac{1}{\tilde{T}}O_P(\frac{\tilde{T}}{\min(N,T^2)})^{\frac{1}{2}}O_P(\tilde{T})^{\frac{1}{2}} = o_p(1)$.

**Proof of Theorem 1:**

$$\hat{V}_{ii}^{-1}\widehat{LM}_i = \frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s'\hat{V}_{i.}^{-1}X_s\hat{\Sigma}_F^{-1}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s\hat{V}_{i.}^{-1}X_s \quad (I)$$
$$- \frac{2}{\tilde{T}^2}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s'\hat{V}_{i.}^{-1}X_s\hat{\Sigma}_F^{-1}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s\hat{F}_s')(\sum_{s=T_a+1}^{T}\hat{F}_s\hat{F}_s')^{-1}\sum_{s=T_a+1}^{T}\hat{F}_s\hat{V}_{i.}^{-1}X_s \quad (II)$$
$$+ \frac{1}{\tilde{T}^2}\sum_{s=T_a+1}^{T}\hat{F}_s'\hat{V}_{i.}^{-1}X_s(\sum_{s=T_a+1}^{T}\hat{F}_s\hat{F}_s')^{-1}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s\hat{F}_s'\hat{\Sigma}_F^{-1}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s\hat{F}_s')$$
$$(\sum_{s=T_a+1}^{T}\hat{F}_s\hat{F}_s')^{-1}\sum_{s=T_a+1}^{T}\hat{F}_s\hat{V}_{i.}^{-1}X_s \quad (III)$$

First consider $(I)$, which can be rewritten as $\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\sum_{m=0}^{4}B_{ms}'\sum_{j\in J_{i,N}}(V_{ij}^{-1}+o_p(1))X_{js}[\tilde{\Sigma}_F^{-1}+o_P(1)]\sum_{s=T_a+1}^{T_a+t}\sum_{n=0}^{4}B_{ns}\sum_{j\in J_{i,N}}(V_{ij}^{-1}+o_p(1))X_{js}$. Similar to the proof of Lemma 6, we focus on the $mn$th ($n\neq 0$) bias term $\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B_{ms}'V_{ij}^{-1}X_{js}\tilde{\Sigma}_F^{-1}\sum_{s=T_a+1}^{T_a+t}B_{ns}V_{ij}^{-1}X_{js}$ , which is bounded by

$||\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}\sum_{s=T_a+1}^{T_a+t}B_{ns}V_{ij}^{-1}X_{js}||||\tilde{\Sigma}_F^{-1}||$ and

$$
\begin{aligned}
& ||\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}\sum_{s=T_a+1}^{T_a+t}B_{ns}V_{ij}^{-1}X_{js}|| \\
\leq & \frac{1}{\tilde{T}^2}(\sum_{t=1}^{\tilde{T}}||\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}||^2)^{\frac{1}{2}}(\sum_{t=1}^{\tilde{T}}||\sum_{s=T_a+1}^{T_a+t}B_{ns}V_{ij}^{-1}X_{js}||^2)^{\frac{1}{2}} \\
\leq & \frac{1}{\tilde{T}^2}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}||B'_{ms}||^2\sum_{s=1}^{t}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}||B_{ns}||^2\sum_{s=1}^{t}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}} \\
\leq & \frac{1}{\tilde{T}}(\sum_{s=T_a+1}^{T}||B'_{ms}||^2\sum_{s=T_a+1}^{T}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}}(\sum_{s=T_a+1}^{T}||B_{ns}||^2\sum_{s=T_a+1}^{T}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}} \\
= & \frac{1}{\tilde{T}}(O_p(\tilde{T})O_p(\tilde{T}))^{\frac{1}{2}}(O_P(\frac{\tilde{T}}{\min(N,T^2)})O_p(\tilde{T}))^{\frac{1}{2}} \\
= & O_P(\frac{\tilde{T}}{\sqrt{\min(N,T^2)}})
\end{aligned}
$$

The first identity follows from Lemmas 2-5, as the dominant bias term is the 02th term or the 04th term depending on $\min(N,T^2)$.

Now consider $(II)$. Due to Lemmas 1 and 6, we only need to study the term $\frac{1}{\tilde{T}^2}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\hat{F}'_s\hat{V}_{i.}^{-1}X_s\hat{\Sigma}_F^{-1}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s\hat{F}'_s)$ which is rewritten as $\frac{1}{\tilde{T}^2}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\sum_{m=0}^{4}B'_{ms}\sum_{j\in J_{i,N}}(V_{ij}^{-1}+o_p(1))X_{js}[\tilde{\Sigma}_F^{-1}+o_P(1)]\sum_{s=T_a+1}^{T_a+t}\sum_{n=0}^{4}B_{ns}\sum_{z=0}^{4}B'_{zs})$.

The $mnz$th ($z\neq 0$) bias term is bounded by $||\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}\sum_{s=T_a+1}^{T_a+t}B_{ns}B'_{zs}||||\tilde{\Sigma}_F^{-1}||$ and by Lemmas 2-5

$$
\begin{aligned}
& ||\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}\sum_{s=T_a+1}^{T_a+t}B_{ns}B'_{zs}|| \\
\leq & \frac{1}{\tilde{T}}(\sum_{s=T_a+1}^{T}||B'_{ms}||^2\sum_{s=T_a+1}^{T}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}}(\sum_{s=T_a+1}^{T}||B_{ns}||^2\sum_{s=T_a+1}^{T}||B'_{zs}||^2)^{\frac{1}{2}} \\
= & \frac{1}{\tilde{T}}(O_p(\tilde{T})O_p(\tilde{T}))^{\frac{1}{2}}(O_P(\frac{\tilde{T}}{\min(N,T^2)})O_p(\tilde{T}))^{\frac{1}{2}} \\
= & O_P(\frac{\tilde{T}}{\sqrt{\min(N,T^2)}})
\end{aligned}
$$

Finally consider $(III)$. Due to Lemmas 1 and 6, we only need to study the term

$$
\begin{aligned}
\frac{1}{\tilde{T}^2}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s\hat{F}'_s\hat{\Sigma}_F^{-1}\sum_{s=T_a+1}^{T_a+t}\hat{F}_s\hat{F}'_s) = & \frac{1}{\tilde{T}^2}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}\sum_{m=0}^{4}B_{ms}\sum_{n=0}^{4}B'_{ns})[\tilde{\Sigma}_F^{-1}+o_P(1)] \\
& \sum_{s=T_a+1}^{T_a+t}\sum_{z=0}^{4}B_{zs}\sum_{l=0}^{4}B'_{ls})
\end{aligned}
$$

The $mnzl$th ($l\neq 0$) bias term is bounded by $||\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B_{ms}B'_{ns}\sum_{s=T_a+1}^{T_a+t}B_{zs}B'_{ls}||||\tilde{\Sigma}_F^{-1}||$ and by Lemmas 2-5

$$
\begin{aligned}
& ||\frac{1}{\tilde{T}^2}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B_{ms}B'_{ns}\sum_{s=T_a+1}^{T_a+t}B_{zs}B'_{ls}|| \\
\leq & \frac{1}{\tilde{T}}(\sum_{s=T_a+1}^{T}||B_{ms}||^2\sum_{s=T_a+1}^{T}||B'_{ns}||^2)^{\frac{1}{2}}(\sum_{s=T_a+1}^{T}||B_{zs}||^2\sum_{s=T_a+1}^{T}||B'_{ls}||^2)^{\frac{1}{2}} \\
= & \frac{1}{\tilde{T}}(O_p(\tilde{T})O_p(\tilde{T}))^{\frac{1}{2}}(O_P(\frac{\tilde{T}}{\min(N,T^2)})O_p(\tilde{T}))^{\frac{1}{2}} \\
= & O_P(\frac{\tilde{T}}{\sqrt{\min(N,T^2)}})
\end{aligned}
$$

The proof is completed by Assumption 4(h)iii.

## .2 Proof of Theorem 2

Different from the other parts in the paper, $\hat{F}$ in this section is $T$ times the eigenvectors corresponding to the $k$ largest eigenvalues of the matrix $XX'$ as in Bai (2004). Let $\tau = \frac{T}{\sqrt{\tilde{T}^2+T}} \in [1, \sqrt{T}]$ and $\varphi_{NT} = min(\sqrt{N}\tau, T)$. The proof is similar to that for Theorem 1.

$$\hat{F}_t - H'F_t = V_{NT}^{-1}(\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\zeta_{st} + \frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\eta_{st} + \frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\xi_{st} + \frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\gamma_{st})$$
$$\hat{F}_t = B_{0t} + B_{1t} + B_{2t} + B_{3t} + B_{4t}$$

**Lemma 9.** $\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s - H'F_s||^2 = \varphi_{NT}^{-2}$.

**Lemma 10.** $\sum_{t=T_a+1}^{T}||\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\zeta_{st}||^2 = O_P(\frac{\tilde{T}}{NT\tau^2})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{1t}||^2 = O_P(\frac{\tilde{T}}{NT\tau^2})$

**Lemma 11.** $\sum_{t=T_a+1}^{T}||\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\eta_{st}||^2 = O_P(\frac{\tilde{T}}{N\tau^4})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{2t}||^2 = O_P(\frac{\tilde{T}}{N\tau^4})$

**Lemma 12.** $\sum_{t=T_a+1}^{T}||\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\xi_{st}||^2 = O_P(\frac{\tilde{T}^2}{NT\tau^2})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{3t}||^2 = O_P(\frac{\tilde{T}^2}{NT\tau^2})$

**Lemma 13.** $\sum_{t=T_a+1}^{T}||\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\gamma_{st}||^2 = O_p(\frac{\tilde{T}}{T^3}\varphi_{NT}^{-2}) + O_p(\frac{\tilde{T}^3}{T^4})$ *i.e.,* $\sum_{t=T_a+1}^{T}||B_{4t}||^2 = O_p(\frac{\tilde{T}}{T^3}\varphi_{NT}^{-2}) + O_p(\frac{\tilde{T}^3}{T^4})$

**Lemma 14.** $\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\hat{F}_t\hat{V}_{i.}^{-1}X_t = \frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\tilde{F}_tV_{i.}^{-1}X_t + o_p(1)$ *when* $N, T \to \infty$.

**Lemma 15.** $\frac{1}{T_a}\sum_{t=1}^{T_a}\hat{F}_t\hat{F}_t' = \frac{1}{T_a}\sum_{t=1}^{T_a}\tilde{F}_t\tilde{F}_t' + o_P(1) = \tilde{\Sigma}_F + o_P(1)$. *Therefore,* $(\frac{1}{T_a}\sum_{t=1}^{T_a}\hat{F}_t\hat{F}_t')^{-1} = \tilde{\Sigma}_F^{-1} + o_P(1)$.

**Lemma 16.** $\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\hat{F}_t\hat{F}_t' = \frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\tilde{F}_t\tilde{F}_t' + o_P(1)$, *when* $N, T \to \infty$.

**Proof of Lemma 9 (Similar to that in Bai (2003) or Bai (2004), omitted)**
**Proof of Lemma 10**

$$
\begin{aligned}
\sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\zeta_{st}||^2 &= \sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s + H'F_s)\zeta_{st}||^2 \\
&\leq \sum_{t=T_a+1}^{T} (||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\zeta_{st}|| + ||\frac{1}{T^2}\sum_{s=1}^{T}H'F_s\zeta_{st}||)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\zeta_{st}||^2 + \sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}H'F_s\zeta_{st}||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s - H'F_s||^2 \frac{1}{NT^3}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}[N^{-\frac{1}{2}}\sum_{i=1}^{N}(u_{is}u_{it} - \mathrm{E}(u_{is}u_{it}))]^2 \\
&\quad + \frac{1}{NT^3}\sum_{t=T_a+1}^{T} ||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N}F_s(u_{is}u_{it} - \mathrm{E}(u_{is}u_{it}))||^2 ||H'||^2) \\
&\leq O_p(\frac{\tilde{T}}{NT^2}\varphi_{NT}^{-2}) \\
&\quad + \frac{2}{NT^4}\sum_{s=1}^{T}||F_s||^2 \sum_{t=T_a+1}^{T}\sum_{s=1}^{T}[N^{-\frac{1}{2}}\sum_{i=1}^{N}(u_{is}u_{it} - \mathrm{E}(u_{is}u_{it}))]^2 ||H'||^2) \\
&= O_p(\frac{\tilde{T}}{NT^2}\varphi_{NT}^{-2}) + O_p(\frac{\tilde{T}}{NT\tau^2}) \\
&= O_p(\frac{\tilde{T}}{NT\tau^2})
\end{aligned}
$$

**Proof of Lemma 11**

$$\begin{aligned}
\sum_{t=T_a+1}^{T} \|\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\eta_{st}\|^2 &= \sum_{t=T_a+1}^{T} \|\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s + H'F_s)\eta_{st}\|^2 \\
&\leq \sum_{t=T_a+1}^{T} (\|\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\eta_{st}\| + \|\frac{1}{T^2}\sum_{s=1}^{T} H'F_s\eta_{st}\|)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T} \|\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\eta_{st}\|^2 + \sum_{t=T_a+1}^{T} \|\frac{1}{T^2}\sum_{s=1}^{T} H'F_s\eta_{st}\|^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}\|\hat{F}_s - H'F_s\|^2 \frac{1}{N^2T^3}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}(F_s'\Lambda' u_t)^2 \\
&\quad + \frac{1}{T^2}\sum_{t=T_a+1}^{T}\|\frac{1}{NT}\sum_{s=1}^{T}F_sF_s'\Lambda' u_t\|^2\|H'\|^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}\|\hat{F}_s - H'F_s\|^2 \frac{1}{NT^3}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}\|F_s'\|^2\|N^{-\frac{1}{2}}\Lambda' u_t\|^2 \\
&\quad + \|\frac{1}{T^2}\sum_{s=1}^{T}F_sF_s'\|^2\frac{1}{N}\sum_{t=T_a+1}^{T}\|N^{-\frac{1}{2}}\Lambda' u_t\|^2\|H'\|^2) \\
&= O_p(\frac{\tilde{T}}{NT\tau^2}\varphi_{NT}^{-2}) + O_p(\frac{\tilde{T}}{N\tau^4}) \\
&= O_p(\frac{\tilde{T}}{N\tau^4})
\end{aligned}$$

**Proof of Lemma 12**

$$
\begin{aligned}
\sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\xi_{st}||^2 &= \sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s + H'F_s)\xi_{st}||^2 \\
&\leq \sum_{t=T_a+1}^{T} (||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\xi_{st}|| + ||\frac{1}{T^2}\sum_{s=1}^{T}H'F_s\xi_{st}||)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\xi_{st}||^2 + \sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}H'F_s\xi_{st}||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s - H'F_s||^2 \frac{1}{N^2T^3}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}(F_t'\Lambda' u_s)^2 \\
&\quad + \frac{1}{T^2}\sum_{t=T_a+1}^{T}||\frac{1}{NT}\sum_{s=1}^{T}F_s u_s'\Lambda F_t||^2||H'||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s - H'F_s||^2 \frac{1}{NT^3}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}||F_t'||^2||N^{-\frac{1}{2}}\Lambda' u_s||^2 \\
&\quad + \frac{1}{NT^3}\sum_{t=T_a+1}^{T}||F_t||^2||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}F_s u_s'\Lambda||^2||H'||^2) \\
&\leq O_p(\frac{\tilde{T}^2}{NT^2}\varphi_{NT}^{-2}) + \frac{2}{NT^4}\sum_{t=T_a+1}^{T}||F_t||^2\sum_{s=1}^{T}||F_s||^2\sum_{s=1}^{T}||N^{-\frac{1}{2}}u_s'\Lambda||^2||H'||^2 \\
&= O_p(\frac{\tilde{T}^2}{NT^2}\varphi_{NT}^{-2}) + O_p(\frac{\tilde{T}^2}{NT\tau^2}) \\
&= O_p(\frac{\tilde{T}^2}{NT\tau^2})
\end{aligned}
$$

**Proof of Lemma 13**

$$
\begin{aligned}
\sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}\hat{F}_s\gamma_{st}||^2 &= \sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s + H'F_s)\gamma_{st}||^2 \\
&\leq \sum_{t=T_a+1}^{T} (||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\gamma_{st}|| + ||\frac{1}{T^2}\sum_{s=1}^{T}H'F_s\gamma_{st}||)^2 \\
&\leq 2(\sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}(\hat{F}_s - H'F_s)\gamma_{st}||^2 + \sum_{t=T_a+1}^{T} ||\frac{1}{T^2}\sum_{s=1}^{T}H'F_s\gamma_{st}||^2) \\
&\leq 2(\frac{1}{T}\sum_{s=1}^{T}||\hat{F}_s - H'F_s||^2 \frac{1}{N^2T^3}\sum_{t=T_a+1}^{T}\sum_{s=1}^{T}[\sum_{i=1}^{N}\mathrm{E}(u_{is}u_{it})]^2 \\
&\quad + \frac{1}{NT^3}\sum_{t=T_a+1}^{T}||\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N}F_s\mathrm{E}(u_{is}u_{it})||^2||H'||^2) \\
&\leq O_p(\frac{\tilde{T}}{T^3}\varphi_{NT}^{-2}) + O_p(\frac{\tilde{T}^3}{T^4})
\end{aligned}
$$

**Proof of Lemma 14**

$$\begin{aligned}\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\hat{F}_t\hat{V}_{i.}^{-1}X_t &= \frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}\hat{V}_{ij}^{-1}X_{jt}\\ &= \frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}(V_{ij}^{-1}+o_p(1))X_{jt}\end{aligned}$$

Note that $\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\tilde{F}_tV_{i.}^{-1}X_t = \frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}B_{0t}\sum_{j\in J_{i,N}}V_{ij}^{-1}X_{jt}$. The order of $\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}V_{ij}^{-1}X_{jt}$ is higher than that of $\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}\sum_{m=0}^{4}B_{mt}\sum_{j\in J_{i,N}}o_p(1)X_{jt}$, since the number of elements in $J_{i,N}$ is finite almost surely when $N\to\infty$. Thus we focus on the $m$th ($m\neq 0$) bias term $\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}B_{mt}V_{ij}^{-1}X_{jt}$ which is is bounded as

$$||\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}B_{mt}V_{ij}^{-1}X_{jt}|| \leq \frac{1}{\tilde{T}^2}(\sum_{t=T_a+1}^{T}||B_{mt}||^2)^{\frac{1}{2}}(\sum_{t=T_a+1}^{T}(V_{ij}^{-1}X_{jt})^2)^{\frac{1}{2}}$$

The dominant bias term is the 3th term or the 4th term depending on $\min(N\tau^2,\frac{T^3}{\tilde{T}})$, i.e. $||\frac{1}{\tilde{T}^2}\sum_{t=T_a+1}^{T}B_{mt}V_{ij}^{-1}X_{jt}|| \leq \frac{1}{\tilde{T}^2}O_P(\frac{\tilde{T}^2}{\min(N\tau^2,\frac{T^3}{\tilde{T}})T})^{\frac{1}{2}}O_P(\tilde{T}^2)^{\frac{1}{2}} = o_p(1)$.

**Proof of Lemma 15**

$\frac{1}{T_a}\sum_{t=1}^{T_a}\hat{F}_t\hat{F}_t{}' = \frac{1}{T_a}\sum_{t=1}^{T_a}\sum_{m=0}^{4}B_{mt}\sum_{n=0}^{4}B_{nt}'$.

$$||\frac{1}{T_a}\sum_{t=1}^{T_a}B_{mt}B_{nt}'|| \leq \frac{1}{T_a}(\sum_{t=1}^{T_a}||B_{mt}||^2)^{\frac{1}{2}}(\sum_{t=1}^{T_a}||B_{nt}||^2)^{\frac{1}{2}}$$

Similar to Lemmas 10-13,

$$\sum_{t=1}^{T_a}||B_{1t}||^2 = O_p(\frac{T_a}{NT\tau^2})$$

$$\sum_{t=1}^{T_a}||B_{2t}||^2 = O_p(\frac{T_a}{N\tau^4})$$

$$\sum_{t=1}^{T_a}||B_{3t}||^2 = O_p(\frac{T_a}{NT\tau^2})$$

$$\sum_{t=1}^{T_a}||B_{4t}||^2 = O_p(\frac{T_a}{T^3}\varphi_{NT}^{-2}) + O_p(\frac{T_a^2}{T^4})$$

The dominant bias term is the 02th term or the 04th term depending on $\min(N\tau^2,\frac{T^4}{T_a\tau^2})$, i.e. $O_P(\frac{1}{\min(N\tau^2,\frac{T^4}{T_a\tau^2})\tau^2})^{\frac{1}{2}}$.

**Proof of Lemma 16 (similar to Lemma 14, omitted)**

**Proof of Theorem 2:**

$(I)$

$$
\begin{aligned}
& ||\frac{1}{\tilde{T}^3}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}\sum_{s=T_a+1}^{T_a+t}B_{ns}V_{ij}^{-1}X_{js}|| \\
\leq\ & \frac{1}{\tilde{T}^3}(\sum_{t=1}^{\tilde{T}}||\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}||^2)^{\frac{1}{2}}(\sum_{t=1}^{\tilde{T}}||\sum_{s=T_a+1}^{T_a+t}B_{ns}V_{ij}^{-1}X_{js}||^2)^{\frac{1}{2}} \\
\leq\ & \frac{1}{\tilde{T}^3}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}||B'_{ms}||^2\sum_{s=1}^{t}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}}(\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}||B_{ns}||^2\sum_{s=1}^{t}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}} \\
\leq\ & \frac{1}{\tilde{T}^2}(\sum_{s=T_a+1}^{T}||B'_{ms}||^2\sum_{s=T_a+1}^{T}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}}(\sum_{s=T_a+1}^{T}||B_{ns}||^2\sum_{s=T_a+1}^{T}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}} \\
=\ & \frac{1}{\tilde{T}^2}(O_p(\tilde{T}^2)O_p(\tilde{T}^2))^{\frac{1}{2}}(O_P(\frac{\tilde{T}^2}{\min(N\tau^2,\frac{T^3}{\tilde{T}})T})O_p(\tilde{T}^2))^{\frac{1}{2}} \\
=\ & O_P(\frac{\tilde{T}^2}{\sqrt{\min(N\tau^2,\frac{T^3}{\tilde{T}})T}})
\end{aligned}
$$

$(II)$

$$
\begin{aligned}
& ||\frac{1}{\tilde{T}^3}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B'_{ms}V_{ij}^{-1}X_{js}\sum_{s=T_a+1}^{T_a+t}B_{ns}B'_{zs}|| \\
\leq\ & \frac{1}{\tilde{T}^2}(\sum_{s=T_a+1}^{T}||B'_{ms}||^2\sum_{s=T_a+1}^{T}(V_{ij}^{-1}X_{js})^2)^{\frac{1}{2}}(\sum_{s=T_a+1}^{T}||B_{ns}||^2\sum_{s=T_a+1}^{T}||B'_{zs}||^2)^{\frac{1}{2}} \\
=\ & O_P(\frac{\tilde{T}^2}{\sqrt{\min(N\tau^2,\frac{T^3}{\tilde{T}})T}})
\end{aligned}
$$

$(III)$

$$
\begin{aligned}
& ||\frac{1}{\tilde{T}^3}\sum_{t=1}^{\tilde{T}}\sum_{s=T_a+1}^{T_a+t}B_{ms}B'_{ns}\sum_{s=T_a+1}^{T_a+t}B_{zs}B'_{ls}|| \\
\leq\ & \frac{1}{\tilde{T}^2}(\sum_{s=T_a+1}^{T}||B_{ms}||^2\sum_{s=T_a+1}^{T}||B'_{ns}||^2)^{\frac{1}{2}}(\sum_{s=T_a+1}^{T}||B_{zs}||^2\sum_{s=T_a+1}^{T}||B'_{ls}||^2)^{\frac{1}{2}} \\
=\ & O_P(\frac{\tilde{T}^2}{\sqrt{\min(N\tau^2,\frac{T^3}{\tilde{T}})T}})
\end{aligned}
$$

## .3 Dataset

| Release | Series | Logs | Diff | Filter | Delay |
|---|---|---|---|---|---|
| GFK consumer confidence | Aggregate balance | 0 | 1 | 3 | 0 |
| GFK consumer confidence | How does the financial situation of your household now compare with what it was 12 months ago | 0 | 1 | 3 | 0 |
| GFK consumer confidence | How do you think the financial position of your household will change over the next 12 months | 0 | 1 | 3 | 0 |
| GFK consumer confidence | How do you think the general economic situation in this country has changed over the last 12 months | 0 | 1 | 3 | 0 |
| GFK consumer confidence | How do you think the general economic situation in this country will develop over the next 12 months | 0 | 1 | 3 | 0 |
| GFK consumer confidence | How do you think the level of unemployment will change over the next 12 months? | 0 | 1 | 3 | 0 |
| GFK consumer confidence | Do you think that there is an advantage for people to make major purchases at the present time | 0 | 1 | 3 | 0 |
| GFK consumer confidence | Over the next 12 months how do you think the amount of money you'll spend on major purchases will compare with what you spent over the last 12 months | 0 | 1 | 3 | 0 |
| ONS retail sale | All Retailers (Volume seasonally adjusted) | 1 | 1 | 3 | 1 |
| ONS retail sale | All Retailers (Value seasonally adjusted) | 1 | 1 | 3 | 1 |
| ONS retail sale | Predominantly food stores (Volume seasonally adjusted) | 1 | 1 | 3 | 1 |
| ONS retail sale | Predominantly food stores (Value seasonally adjusted) | 1 | 1 | 3 | 1 |
| ONS retail sale | Non-specialised stores | 1 | 1 | 3 | 1 |
| ONS retail sale | Textiles: clothing: footwear | 1 | 1 | 3 | 1 |
| ONS retail sale | Household goods stores | 1 | 1 | 3 | 1 |
| ONS retail sale | Non-store retailing & repair | 1 | 1 | 3 | 1 |
| CBI distributive trades | Retailing Sales | 0 | 1 | 3 | 0 |
| CBI distributive trades | Retailing Orders | 0 | 1 | 3 | 0 |
| CBI distributive trades | Retailing Sales for Time of Year | 0 | 1 | 3 | 0 |
| CBI distributive trades | Retailing Stocks | 0 | 1 | 3 | 0 |
| CBI distributive trades | Wholesaling Sales | 0 | 1 | 3 | 0 |
| CBI distributive trades | Wholesaling Orders | 0 | 1 | 3 | 0 |
| CBI distributive trades | Wholesaling Sales for Time of Year | 0 | 1 | 3 | 0 |
| CBI distributive trades | Wholesaling Stocks | 0 | 1 | 3 | 0 |
| CBI distributive trades | Motor Traders Sales | 0 | 1 | 3 | 0 |
| CBI distributive trades | Motor Traders Orders | 0 | 1 | 3 | 0 |
| CBI distributive trades | Motor Traders Sales for Time of Year | 0 | 1 | 3 | 0 |
| CBI distributive trades | Motor Traders Stocks | 0 | 1 | 3 | 0 |
| ONS travel | UK visits abroad: Expenditure abroad | 1 | 1 | 3 | 2 |
| ONS travel | OS visits to UK: Earnings | 1 | 1 | 3 | 2 |
| ONS trade | BOP: Balance, sa, Total Trade in Goods | 0 | 1 | 3 | 2 |
| ONS trade | BOP: Balance, Manufactures | 0 | 1 | 3 | 2 |
| ONS trade | BOP: Balance, Intermediate goods | 0 | 1 | 3 | 2 |

| Release | Series | Logs | Diff | Filter | Delay |
|---|---|---|---|---|---|
| ONS trade | BOP: IM: CVM: sa: Total Trade in Goods | 1 | 1 | 3 | 2 |
| ONS trade | BOP: EX: CVM: sa: Total Trade in Goods | 1 | 1 | 3 | 2 |
| ONS trade | BOP: Balance, Capital goods | 0 | 1 | 3 | 2 |
| ONS trade | BOP: IM, price index, Finished manufactures | 1 | 1 | 3 | 2 |
| ONS trade | Balance of Payments: Trade in Services: Total balance: CP | 0 | 1 | 3 | 2 |
| FTSE | All Share Dividend Yield | 0 | 1 | 3 | 0 |
| FTSE | All Share Price / Earnings Ratio | 0 | 1 | 3 | 0 |
| FTSE | All Share Price Index | 1 | 1 | 3 | 0 |
| FTSE | FTSE 100 | 1 | 1 | 3 | 0 |
| Exchange rate | Japanese Yen /£ | 1 | 1 | 3 | 0 |
| Exchange rate | United States Dollar /£ | 1 | 1 | 3 | 0 |
| Exchange rate | Effective exchange rate index, Sterling (Jan 2005=100) | 1 | 1 | 3 | 0 |
| Interest rate | Bank Of England Repo Rate | 0 | 1 | 3 | 0 |
| Interest rate | Overnight £ Inter-Bank Rate (Mean Libid/Libor)–8.30am | 0 | 1 | 3 | 0 |
| Interest rate | ICE LIBOR GBP 3 Month | 0 | 1 | 3 | 0 |
| Interest rate | 3 Month £ Inter-Bank Rate (Mean Libid/Libor)–10.30am | 0 | 1 | 3 | 0 |
| Interest rate | 6 Month £ Inter-Bank Rate (Mean Libid/Libor)–8.30am | 0 | 1 | 3 | 0 |
| Interest rate | Monthly average yield from British Government Securities, 20 years | 0 | 1 | 3 | 0 |
| Interest rate | End month level of discount rate, 3 month Treasury bills | 0 | 1 | 3 | 0 |
| Volatility | LIFFE FTSE 100 3 Months Constant Maturity: Implied Volatility | 0 | 1 | 3 | 0 |
| VRP | VRPSPOT(NOM,UK,5) | 0 | 1 | 3 | 0 |
| VRP | VRPSPOT(NOM,UK,10) | 0 | 1 | 3 | 0 |
| VRP | VRPSPOT(REL,UK,5) | 0 | 1 | 3 | 0 |
| VRP | VRPSPOT(REL,UK,10) | 0 | 1 | 3 | 0 |
| VRP | VRPSPOT(INF,UK,5) | 0 | 1 | 3 | 0 |
| VRP | VRPSPOT(INF,UK,10) | 0 | 1 | 3 | 0 |
| ONS labor | Total Claimant count | 0 | 1 | 3 | 1 |
| ONS labor | Claimant count rate | 0 | 1 | 3 | 1 |
| ONS labor | AEI (including bonuses), whole economy | 1 | 1 | 3 | 2 |
| ONS labor | AEI (including bonuses), private sector | 1 | 1 | 3 | 2 |
| ONS labor | In employment: UK: All: Aged 16+ | 1 | 1 | 3 | 3 |
| ONS labor | Unemployed: UK: All: Aged 16+ | 1 | 1 | 3 | 3 |
| ONS labor | Total actual weekly hours worked (millions): UK: All | 1 | 1 | 3 | 3 |
| ONS labor | Unemployed up to 6 months: UK: All: Aged 16+ | 1 | 1 | 3 | 3 |
| ONS labor | Unemployed over 6 and up to 12 months: UK: All: Aged 16+ | 1 | 1 | 3 | 3 |
| ONS labor | Unemployed over 12 months: UK: All: Aged 16+ | 1 | 1 | 3 | 3 |
| ONS labor | Unemployed over 24 months: UK: All: Aged 16+ | 1 | 1 | 3 | 3 |
| MST | Monthly amounts outstanding of monetary financial institutions' sterling M4 liabilities to private sector | 1 | 1 | 3 | 1 |
| MST | Monthly average amount outstanding of total sterling notes and coin in circulation out-side the Bank of England | 1 | 1 | 3 | 1 |

| Release | Series | Logs | Diff | Filter | Delay |
|---|---|---|---|---|---|
| MST | VQWK.M | 1 | 1 | 3 | 1 |
| MST | Money Stock: Retail Deposits and Cash in M4: NSA | 1 | 1 | 3 | 1 |
| MST | Monthly amounts outstanding of monetary financial institutions' sterling net lending to private sector | 1 | 1 | 3 | 1 |
| MST | Monthly value of total sterling approvals for secured lending to individuals | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Consumer Goods Industries - Total New Orders | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Supplier's Delivery Times | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Employment | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Stocks Of Finished Goods | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Investment Goods Industries | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Total New Orders | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Output | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Quantity Of Purchases | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Stocks Of Purchases | 1 | 1 | 3 | 1 |
| CIPS manufacturing | Input Prices | 1 | 1 | 3 | 1 |
| ONS output | Industry C,D,E: All production industries | 1 | 1 | 3 | 2 |
| ONS output | Industry C: Mining & quarrying | 1 | 1 | 3 | 2 |
| ONS output | Industry D: Manufacturing | 1 | 1 | 3 | 2 |
| ONS output | Industry E: Electricity, gas and water supply | 1 | 1 | 3 | 2 |
| ONS output | Industry DA: Manuf of food, drink & tobacco | 1 | 1 | 3 | 2 |
| ONS output | Industry DB: Manuf of textile & textile products | 1 | 1 | 3 | 2 |
| ONS output | Industry DC: Manuf of leather & leather products | 1 | 1 | 3 | 2 |
| ONS output | Industry DD: Manuf of wood & wood products | 1 | 1 | 3 | 2 |
| ONS output | Industry DE: Pulp/paper/printing/publishing industries | 1 | 1 | 3 | 2 |
| ONS output | Industry DF: Manuf coke/petroleum prod/nuclear fuels | 1 | 1 | 3 | 2 |
| ONS output | Industry DG: Manuf of chemicals & man-made fibres | 1 | 1 | 3 | 2 |
| ONS output | Industry DH: Manuf of rubber & plastic products | 1 | 1 | 3 | 2 |
| ONS output | Industry DI: Manuf of non-metallic mineral products | 1 | 1 | 3 | 2 |
| ONS output | Industry DJ: Manuf of basic metals & fabricated prod | 1 | 1 | 3 | 2 |
| ONS output | Industry DK: Manuf of machinery & equipment | 1 | 1 | 3 | 2 |
| ONS output | Industry DL: Manuf of electrical & optical equipment | 1 | 1 | 3 | 2 |
| ONS output | Industry DM: Manuf of transport equipment | 1 | 1 | 3 | 2 |
| CBI monthly trends | Do you consider that in vo-lume terms, your present total order book is above normal? | 0 | 1 | 3 | 0 |
| CBI monthly trends | Do you consider that in volume terms, your present export order book is above normal? | 0 | 1 | 3 | 0 |
| CBI monthly trends | Adequacy of Stocks of Finished Goods | 0 | 1 | 3 | 0 |
| CBI monthly trends | What is the expected trend over the next 4 months with regards to your volume of output? | 0 | 1 | 3 | 0 |
| CBI monthly trends | What is the expected trend over the next 4 months with regards to average prices for domestic orders? | 0 | 1 | 3 | 0 |
| Experian Construction | United Kingdom: Experian Construction Activity (Index) | 1 | 1 | 3 | 1 |
| Brent crude | 1 Month Fwd, fob U$/BBL | 1 | 1 | 3 | 0 |

| Release | Series | Logs | Diff | Filter | Delay |
|---|---|---|---|---|---|
| Brent crude | Physical Del., fob U$/BBL | 1 | 1 | 3 | 0 |
| ONS PPI | NSI: All manufacturing: Materials only | 1 | 2 | 3 | 1 |
| ONS PPI | Prod of man ind excl.f,b, p & t sa | 1 | 2 | 3 | 1 |
| ONS PPI | Fuels Purchased by Man Ind Excl CCL | 1 | 2 | 3 | 1 |
| ONS PPI | NSI: M & F purchased by Man: Excl FBPT Excl CCL NSA | 1 | 2 | 3 | 1 |
| ONS PPI | Output of manufactured products | 1 | 2 | 3 | 1 |
| ONS PPI | NSO: All Manufacturing excl duty: sa | 1 | 2 | 3 | 1 |
| ONS CPI | Food And Non-Alcoholic Beverages | 1 | 2 | 12 | 1 |
| ONS CPI | Alcoholic Beverages, Tobacco & Narcotics | 1 | 2 | 12 | 1 |
| ONS CPI | Clothing And Footwear | 1 | 2 | 12 | 1 |
| ONS CPI | Housing, Water And Fuels | 1 | 2 | 12 | 1 |
| ONS CPI | Furn, Hh Equip & Routine Repair Of House | 1 | 2 | 12 | 1 |
| ONS CPI | Health | 1 | 2 | 12 | 1 |
| ONS CPI | Transport | 1 | 2 | 12 | 1 |
| ONS CPI | Communication | 1 | 2 | 12 | 1 |
| ONS CPI | Recreation & Culture | 1 | 2 | 12 | 1 |
| ONS CPI | Education | 1 | 2 | 12 | 1 |
| ONS CPI | Hotels, Cafes And Restaurants | 1 | 2 | 12 | 1 |
| ONS CPI | Miscellaneous Goods And Services | 1 | 2 | 12 | 1 |
| HBF | Change in net house prices during the month | 0 | 1 | 3 | 2 |
| HBF | Site Visits, compared with a year ago | 0 | 1 | 3 | 2 |
| HBF | Net Reservations, compared with a year ago | 0 | 1 | 3 | 2 |
| HAC | RICS Housing Market Survey, Prices, England and Wales, Net Balance | 0 | 1 | 3 | 1 |
| HAC | Ratio Of RICS Sales Series To RICS Stock Series | 0 | 1 | 3 | 1 |
| US | VIX | 0 | 1 | 3 | 0 |
| US | TED | 0 | 1 | 3 | 0 |
| US | Manufacturing PMI | 1 | 1 | 3 | 1 |
| US | IP manufacturing | 1 | 1 | 3 | 1 |
| US | EXPORTS F.A.S. CURA | 1 | 1 | 3 | 2 |
| US | IMPORTS F.A.S. CURA | 1 | 1 | 3 | 2 |
| US | Retail sales (current prices) | 1 | 1 | 3 | 1 |
| US | S&P/CASE-SHILLER HOME PRICE INDEX - 10-CITY COMPOSITE SADJ | 1 | 2 | 3 | 2 |
| US | Employment | 1 | 1 | 3 | 1 |
| US | Unemployment rate | 0 | 1 | 3 | 1 |
| US | M2 | 1 | 1 | 3 | 1 |
| US | S&P monthly average | 0 | 1 | 3 | 0 |
| Germany | Assessment of business | 1 | 1 | 3 | 0 |
| Germany | bussiness expectation | 1 | 1 | 3 | 0 |
| Germany | consumer confidence | 0 | 1 | 3 | 0 |
| Germany | Export | 1 | 1 | 3 | 2 |
| Germany | Import | 1 | 1 | 3 | 2 |
| Germany | IP manufacturing | 1 | 1 | 3 | 1 |

## References


Andrews, D.W.K, 2003, End-of-sample instability tests, Econometrica 71(6), 1661–1694.

Angelini, Elena, Gonzalo Camba-Mendez, Domenico Giannone, Lucrezia Reichlin, and Gerhard Rünstler, 2011, Short-term forecasts of euro area gdp growth, The Econometrics Journal 14(1), C25–C44.

Bai, Jushan, 2003, Inferential theory for factor models of large dimensions, Econometrica 71(1), 135–171.

_ 2004, Estimating cross-section common stochastic trends in nonstationary panel data, Journal of Econometrics 122(1), 137–183.

Bai, Jushan, and Serena Ng, 2002, Determining the number of factors in approximate factor models, Econometrica 70(1), 191–221.

_ 2008a, Forecasting economic time series using targeted predictors, Journal of Econometrics 146(2), 304–317.

_ 2008b, Large dimensional factor analysis, Foundations and Trends (R) in Econometrics 3(2), 89–163.

_ 2009, Boosting diffusion indices, Journal of Applied Econometrics 24(4), 607–629.

Bai, Jushan, and Yuan Liao, 2013, Efficient estimation of approximate factor models via regularized maximum likelihood.

Bates, Brandon J, Mikkel Plagborg-Møller, James Stock, and Mark Watson, 2013, Consistent factor estimation in dynamic factor models with structural instability, Journal of Econometrics 177(2), 289–304.

Boivin, Jean, and Serena Ng, 2006, Are more data always better for factor analysis?, Journal of Econometrics 132(1), 169–194.

Breitung, Jörg, and Sandra Eickmeier, 2011, Testing for structural breaks in dynamic factor models, Journal of Econometrics 163(1), 71–84.

Chen, Liang, Juan J Dolado, and Jesús Gonzalo, 2014, Detecting big structural breaks in large factor models, Journal of Econometrics 180(1), 30–48.

Cheng, Xu, Zhipeng Liao, and Frank Schorfheide, 2014, Shrinkage estimation of high-dimensional factor models with structural instabilities, National Bureau of Economic Research.

Choi, In, 2012, Efficient estimation of factor models, Econometric Theory 28(02), 274–308.

Corradi, Valentina, and Norman R Swanson, 2014, Testing for structural stability of factor augmented forecasting models, Journal of Econometrics.

Del Negro, Marco, and Christopher Otrok, 2008, Dynamic factor models with time-varying parameters: measuring changes in international business cycles.

Fan, Jianqing, Yuan Liao, and Martina Mincheva, 2013, Large covariance estimation by thresholding principal orthogonal complements, Journal of the Royal Statistical Society: Series B (Statistical Methodology) 75(4), 603–680.

Giannone, Domenico, Lucrezia Reichlin, and David Small, 2008, Nowcasting: The real-time informational content of macroeconomic data, Journal of Monetary Economics 55(4), 665–676.

Han, Xu, and Atsushi Inoue, forthcoming, Tests for parameter instability in dynamic factor models, Econometric Theory.

Hansen, Bruce E, 1992, Tests for parameter instability in regressions with i (1) processes, Journal of Business & Economic Statistics 10(3), 45–59.

Nyblom, Jukka, 1989, Testing for the constancy of parameters over time, Journal of the American Statistical Association 84(405), 223–230.

Onatski, Alexei, 2010, Determining the number of factors from empirical distribution of eigenvalues, The Review of Economics and Statistics 92(4), 1004–1016.

Stock, James, and Mark Watson, 1999, Forecasting inflation, Journal of Monetary Economics 44(2), 293–335.

_ 2009, Forecasting in dynamic factor models subject to structural instability, The Methodology and Practice of Econometrics. A Festschrift in Honour of David F. Hendry 173–205.

_ 2011, Dynamic factor models, Oxford Handbook of Economic Forecasting 1, 35–59.

Takongmo, Charles Olivier Mao, and Dalibor Stevanovic, 2014, Selection of the number of factors in presence of structural instability: a monte carlo study.

Yamamoto, Yohei, and Shinya Tanaka, 2013, Testing for factor loading structural change under common breaks.

Yamazaki, Daisuke, and Eiji Kurozumi, 2014, Testing for parameter constancy in the time series direction in fixed-effect panel data models, Journal of Statistical Computation and Simulation.