Growth cycles: Theory and evidence of determinants of the output growth rate.

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Abstract

This paper presents DSGE growth cycle theory and econometric evidence of determinants of the output growth rate. For both a baseline and extended DSGE monetary model, moments are matched across a range of *RBC* and monetary moments over both business cycle and lower frequencies using US data. Econometric testing is then conducted with a model based on the theoretical effects found in the DSGE model. We test a small panel of OECD countries by applying a recent panel technique to growth estimation called Small-N, for a small number of countries, as selected only by data availability. We find that the inflation tax remains a solid negative effect. This result is enhanced by including the capacity utilization rate as urged in Plosser's (2000) real business cycle theory. This capital utilization effect is significant and positive as in our DSGE business cycle theory. In addition we include taxes more generally than just the inflation tax, in the style of Ohanian and Rogerson (2009), and find negatively significant the effect of a combined tax on labor and capital income, as our DSGE model also supports.

JEL: C23, E44, O16, O42

Keywords: Inflation, growth, panel data, capacity utilization, taxes.

1. Introduction

A robust negative effect of inflation on growth has been found in many studies, with the only debate still lingering at low levels of inflation.[1] The negative effect makes sense in that inflation is a tax that decreases growth in the standard monetary extensions of the Lucas (1988) endogenous growth economy. This negative effect is found in evidence such as Barro (1990), and in theory as in Gomme (1993) and Gillman and Kejak (2005,2011) and in combined theory and evidence, eg. Gillman et al. (2004), Benk et al. (2008, 2010) and Basu et al. (2012). A possible positive correlation of inflation and growth can result during business cycles since an expansion see output rising and productivity shocks cause a higher price of output relative to labor which gets recorded as "inflation" increases; but here the causality can be from growth to measured inflation.

As the growth literature has extended to include "growth cycles" that weave together endogenous growth with real business cycle (RBC) theory, long run growth can be enhanced theoretically by incorporating certain key business cycle elements. Then the model can be tested with a comparison of simulations to data moments in RBC fashion over the business cycle frequency as well as over lower frequencies, and Comin and Gertler's (2006) Medium Term frequency. Empirically, panel methods can filter out high frequency effects and focus on the longer term as well.

This paper presents such growth cycle *DSGE* theory, comparison of US data moments to *DSGE* simulations, and international econometric evidence of determinants of the output growth rate. Moments are matched across a range of *RBC* and monetary moments over both business cycle and lower frequencies. Panel data testing is conducted for an econometric model based on the theoretical effects found in the DSGE model. We test a panel of 12 European Union countries,

[1] "A little inflation is good" is a deep historical macroeconomic issue supported in a sequence of Keynesian flavored models from the first output gap of Samuelson (1951) to the current New-Keynesian *DSGE* model's with monopolistic competition and sticky prices; but it remains far from accepted that econometric evidence supports low levels of an inflation tax positively affects output growth in the long run. good.

including one "transition country", plus the US, all selected only by data availability. We find that the inflation tax remains a solid negative effect. This result is enhanced by including the capacity utilization rate as urged in Plosser's (2000) real business cycle theory. This capital utilization effect is significant and positive as in our DSGE business cycle theory. In addition we include taxes more generally than just the inflation tax, in the style of Ohanian and Rogerson (2009), and find negatively significant the effect of a combined tax on labor and capital income, as our DSGE model also supports. We also extend beyond Gomme (1993), for example, by including a human capital sectoral shock in our $DSGE$ model which enhances its performance and use a proxy of this in the econometric testing.

The econometric methodology is extended from previous work but has not been applied to growth models to our knowledge. The contribution here is application of Small-N panel techniques, where N is the number of countries, whereas the standard panel approach relies on Large-N techniques. This is key here since the ability to include tax rates limits the country sample to a much smaller, more developed set, than if tax rates are not included. Including physical capital capacity utilization also limits the country size. The Small-N approach gives internally consistent, plausible, results. And the results appear to support well the theory.

Empirically explaining international output growth remains a key issue because of the importance of growth in so many nations's public policy agendas. A good empirical explanation can give insight as to how to structure fiscal (income tax rates) and monetary policy (the inflation tax) such that economic growth can be improved efficaciously. The empirical growth literature has developed far since the Kormendi and Meguire (1985) cross-sectional analysis but still remains limited in how its results can enlighten growth policy. Advances in panel estimation have greatly increased the ability to well-explain growth, but advances of bringing in tax effects on growth, for example as has been done for labor market employment by Ohanian and Rogerson (2009), remains limited. Second, the ability to meld the business cycle explanatory advances started by Kydland and Prescott (1982) has not been well-integrated into explaining long run growth, giving rise to the hidden elephant in this literature as to how long run growth is actually to be estimated in a methodologically certain way. Kormendi and Meguire use data points of 5 year averages and that has often been a mainstream approach to growth estimation ever since. Yet it and related methods are arbitrary approaches to estimating long run growth and they leaves business cycle effects to be picked up hopefully through the now prevalent use of fixed effects in panel estimation.

This paper contributes to the growth estimation literature by introducing key policy tax rates into the analysis along with two other features. We bring into the theoretical fold of the estimated growth model the advanced methodology of real business cycle models and we use an econometric approach that well-defines a methodology for capturing the "long run" while treating business cycle effects in a methodologically advanced way as well. On these three advances in particular we introduce in panel estimation for a developed country sample both personal and corporate income tax rates (Stokey and Rebelo, 1996), along with the monetary based inflation tax, going back to its treatment as a key tax in Bailey (1956) and found in growth estimation (Gillman et al. 2005). Second, we present a theoretical model, upon which we form the econometric model, which endogenizes the growth rate, includes both human and physical capital, and includes the capacity utilization rate of both human and physical capital that has played key roles in explaining real and monetary business cycle facts (King, 2000; Benhabib, 1997 ; Benk et al. 2005). The personal and corporate income taxes, and the inflation tax, fall on both human and physical capital so that each of these two main capital returns, and the subsequent theoretical growth rate, are affected by the taxes. In particular, the taxes work through both the marginal product of each capital plus through each capital's capacity utilization rate.

Section 2 presents the full encompassing model. This includes the physical capital utilization rate, along with the human capital utilization rate, and taxes on labor and capital income. Model 1 is then a specified special case, and our "Baseline" model similar to Gomme (1993) with income taxes added. Section

3 provides the simulation results are compared to the data, in terms of correlation moments and volatilities. Section 4 presents the data and specification of the econometric model. Section 5 presents econometric results, Section 6 makes comments and Section 7 concludes. Appendices present model equilibrium conditions, the balanced growth path solution, the $DSGE$ solution methodology, and the $DSGE$ data description.

2. Model with Capacity Utilization, Taxes

In this section the model is presented, which includes a convex endogenous physical capital depreciation rate as a function of the utilization rate to physical capital as in Benhabib and Wen (2003); and the addition of money balances with an exchange technology in line with Stockman (1981). In what we call Model 1, we set capacity utilization to one, and use the standard cash-only Clower (1978) constraint for the model's exchange constraint (the same as Gomme (1993) except that we also have income taxes as well as a human capital sectoral shock). In the following general model specification this Model 1 case is achieved by setting $\chi = \Omega = 0$; this parameters are specified below. We call this Model 1 our "Baseline" model. The full model, below, we then call Model 2.

2.1. The Household's Problem

The representative household maximises the present value of its infinite sum of period utilities, where the period utility function takes the following simple logarithmic form

$$U(c_t, l_t, e_t) = \ln c_t + \alpha \ln x_t + \chi \ln e_t.$$

The household then maximises the above utility function in each time period t subject to the time constraint, where it uses its normalised time endowment to leisure, x_t, work in the goods sector, l_{gt}, or in the human capital investment sector, l_{ht}. Then the time constraint is,

$$1 = x_t + l_{gt} + l_{ht}. \tag{2.1}$$

The household also accumulates human capital over time. Human capital investment is produced by the household according to the following constant returns to scale production function, where the inputs are effective labor, $l_{ht}h_t$, and utilised sectoral physical capital, $s_{ht}u_tk_t$. The human capital investment technology is given by

$$i_{ht} = A_h z_t^h (l_{ht}h_t)^{\eta}(s_{ht}u_tk_t)^{1-\eta}, \tag{2.2}$$

where z_t^h is a sectoral productivity shock process, which is assumed to take the following AR(1) form,

$$z_t^h = \rho_h z_{t-1}^h + \epsilon_t^h, \tag{2.3}$$

where ϵ_t^h is a white noise with constant variance σ_h^2. Human capital is then accumulated according to the following human capital accumulation constraint

$$h_{t+1} = A_h z_t^h (l_{ht}h_t)^{\eta}(s_{ht}u_tk_t)^{1-\eta} + (1-\delta_h)h_t. \tag{2.4}$$

The household also owns physical capital, which is accumulated according to the following standard law of motion with endogenous depreciation rate, which is a convex function of total factor utilisation rate [i.e. proxy of used managerial capital],

$$k_{t+1} = i_{kt} + k_t - \frac{\delta_k}{\psi}u_t^{\psi}k_t. \tag{2.5}$$

Total physical capital in the economy can be freely reallocated across sectors in the beginning of each time period. The share of physical capital in each sector out of the total capital stock must add up to unity,

$$1 = s_{gt} + s_{ht}. \tag{2.6}$$

The agent is also endowed with managerial capacity, from which if it is not used, e_t, it derives utility, and when it is used, u_t, it translates into the utilisation of physical capital economy wide.The endowment of managerial capacity is normalised to one, in which case the managerial capacity constraint is

$$1 = e_t + u_t. \tag{2.7}$$

The agent uses real money balances to buy consumption goods and to finance investment, which is governed by the exchange constraint

$$c_t + \Omega i_{kt} + n_t = m_{t-1} \tag{2.8}$$

One may note, when $\Omega = 0$ the exchange constraint in (8) implies that real money balances are used to purchase consumption only. When $\Omega = 1$ the exchange technology constraint implies that both physical investment goods and consumption can be purchased. For the underlying baseline model Ω is assumed to be one, which scenario has been introduced by Stockman (1981).

The household earns labor income, $w_t l_{gt} h_t$, rents from capital, $r_t s_{gt} u_t k_t$; and income on riskless government bonds held over the previous period, $(1+R_t)b_t$. The household also receives lump-sum government transfers in real terms, n_t, to augment real money balances of the household used in goods and/or investment good purchases. The household's expenditures include consumption, c_t, investment, i_{kt}, and government bond purchases to be held over the given period, $(1 + \pi_{t+1})b_{t+1}$. With tax rates τ_{lt} and τ_{kt}, then the agent faces the following budget constraint in each time period,

$$\begin{aligned} c_t &= w_t (1 - \tau_{lt}) l_{gt} h_t + r_t (1 - \tau_{kt}) s_{gt} u_t k_t - k_{t+1} + k_t && (2.9) \\ & \quad -(1 + \pi_{t+1})b_{t+1} + (1 + R_t)b_t - (1 + \pi_{t+1})m_{t+1} + m_t + n_t && (2.10) \end{aligned}$$

Then the household maximises the expected present value of its lifetime stream of utility,

$$\max_{\{c_t,u_t,l_{gt},l_{ht},s_{gt},s_{ht},k_t,h_{t+1},m_{t+1},b_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_t + \alpha \ln x_t + \chi \ln e_t] \tag{2.11}$$

subject to (1), (2), (4), (5), (6), (7), (8), (9).

Given the household's optimisation problem given in (10) the household faces the following equilibrium conditions,

$$1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right) \left[1 + \frac{r_{t+1}(1-\tau_{kt})\, u_{t+1}}{1+R_{t+1}} - \frac{\delta_k}{\psi} u_{t+1}^{\psi} \right] \right], \tag{2.12}$$

$$1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right) \left(\frac{1+R_t}{1+R_{t+1}} \right) \left(\frac{p_{ht+1}}{p_{ht}} \right) \left[1 + (1-x_{t+1}) r_{t+1}^h - \delta_h \right] \right], \tag{2.13}$$

$$(1+R_t) = E_t \left[\left[1 + \frac{r_{t+1}(1-\tau_{kt})\, u_{t+1}}{1+R_{t+1}} - \frac{\delta_k}{\psi} u_{t+1}^{\psi} \right] (1+\pi_{t+1}) \right], \tag{2.14}$$

$$\frac{\alpha}{x_t} \frac{c_t}{h_t} = \frac{w_t(1-\tau_{lt})}{1+R_t}, \tag{2.15}$$

$$\frac{\chi}{(1-u_t)} \frac{c_t}{k_t} = \frac{r_t(1-\tau_{kt})}{(1+R_t)} - \delta_k u_t^{\psi-1}, \tag{2.16}$$

where p_t^h is the relative price of human capital to consumption goods; r_t is the rental rate of physical capital defined as the marginal product of sectoral utilised physical capital in the firm's problem; and r_t^h is the return on human capital defined as the marginal product of effective labor in the human sector. One can find the complete set of first order conditions for the baseline model including that of the household's, goods producer's, and government's in *Appendix A* along with the complete system of equations of the baseline model.

Equation (11) and (12) are the inter temporal conditions of the household. Equations (14) and (15) are the marginal rate substitution equations between consumption, c_t, and leisure, x_t; and between consumption, c_t, and managerial capacity, u_t. Lastly, equation (13) describes the *Fisher relationship* between the nominal interest rate on bonds and the rate of inflation. Note that as a result of the Stockman (1981) constraint the current nominal interest rate is described by a fully forward looking relationship.

2.2. The Goods Producer's Problem

The goods producer firm maximises profit Π_t, given by

$$\Pi_t = y_t - w_t l_{gt} h_t - r_t s_{gt} u_t k_t, \tag{2.17}$$

subject to a constant returns to scale *Cobb-Douglas* production technology in effective labor, $l_{gt}h_t$, and sectoral utilised physical capital, $s_{gt}u_t k_t$, similar to King and Rebelo (1990):

$$y_t = A_g z_t^g (l_{gt} h_t)^{\gamma} (s_{gt} u_t k_t)^{1-\gamma}. \tag{2.18}$$

The first order conditions for the firm are

$$r_t = (1-\gamma) A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma}, \tag{2.19}$$

$$w_t = \gamma A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma - 1}, \tag{2.20}$$

where z_t^g is a sectoral productivity shock process, which is assumed to take the following AR(1) form,

$$z_t^g = \rho_g z_{t-1}^g + \epsilon_t^g, \tag{2.21}$$

where ϵ_t^g is a white noise with constant variance σ_g^2.

2.3. The Government

The government budget constraint has spending of a lump-sum nominal transfer of cash, N_t, that is given to the household each time period. Given that the nominal output goods price is denoted by P_t, this budget constraint is

$$N_t = M_{t+1} - M_t + P_t \left[(1-\tau_{lt}) \, w_t l_{gt} h_t + (1-\tau_{lt}) \, r_t s_{gt} u_t k_t \right]. \tag{2.22}$$

Assuming that the underlying money supply is such that there is a constant rate of money supply growth along the *balanced growth path*, defined by $\sigma_t \equiv \left\{ N_t - P_t \left[(1-\tau_{lt}) \, w_t l_{gt} h_t + (1-\tau_{lt}) \, r_t s_{gt} u_t k_t \right] \right\} / M_t$, and with $n_t \equiv N_t / P_t$the above money supply can be expressed as,

$$M_{t+1} = M_t(1+\sigma_t). \tag{2.23}$$

In order to be consistent with previous notation the money supply in real terms, the money supply rule becomes

$$(1+\pi_{t+1})m_{t+1} = m_t(1+\sigma_t); \tag{2.24}$$

where σ_t is a stochastic money supply growth process, which is assumed to take the following AR(1) form,

$$\sigma_t = \bar{\sigma} + \rho_m \sigma_{t-1} + \epsilon_t^m, \tag{2.25}$$

where ϵ_t^m is a white noise with constant variance σ_m^2; and $\bar{\sigma}$ is the constant *BGP* rate of money supply growth.

2.4. Characterisation of the Equilibrium

(E.1): Given the processes $\{\pi_{t+1}\}$, $\{w_t\}$, $\{r_t\}$, $\{\tau_{lt}\}$, $\{\tau_{kt}\}$, $\{z_t^h\}$, $\{R_t\}$, and $\{n_t\}$ the household solves its utility maximisation problem in (10).

(E.2): Given the processes $\{w_t\}$, $\{r_t^q\}$, $\{z_t^g\}$, and the production technology in equation (17) the goods producer maximises (16).

(E.3): Asset, goods, and money markets clear so that $b_t = 0$, $n_t = \sigma_t m_{t-1}$, and $y_t = c_t + i_{kt}$.

3. Simulation Results

3.1. Calibration

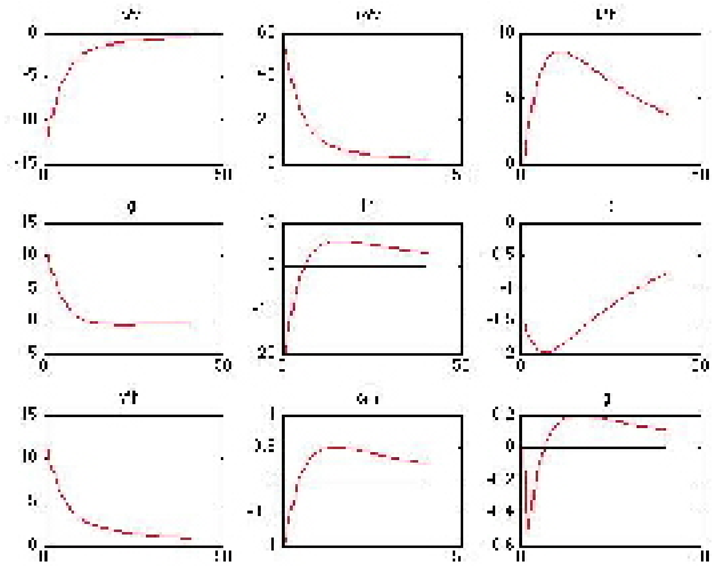
The data series have been filtered by Christiano and Fitzgerald (1999) type asymmetric band-pass filter following Baxter and King (1999), Commin and Gertler (2006), Benk et al. (2010), and Basu et al. (2012). As the US quarterly data, using the period of 1971:2 to 2012:2, is of quarterly frequency the low frequency component has a periodicity of 32 up to 100 quarters, the business cycle component has a periodicity of 6 to 32 quarters, and the high frequency is defined as 2 to 6 quarters in periodicity. The data is filtered by the underlying band pass filter at the medium term frequency which includes all components and has a defined periodicity of 2 up to 150 quarters in line with the definition of Commin and Gertler (2006). Alltogether there are 37 years of data.

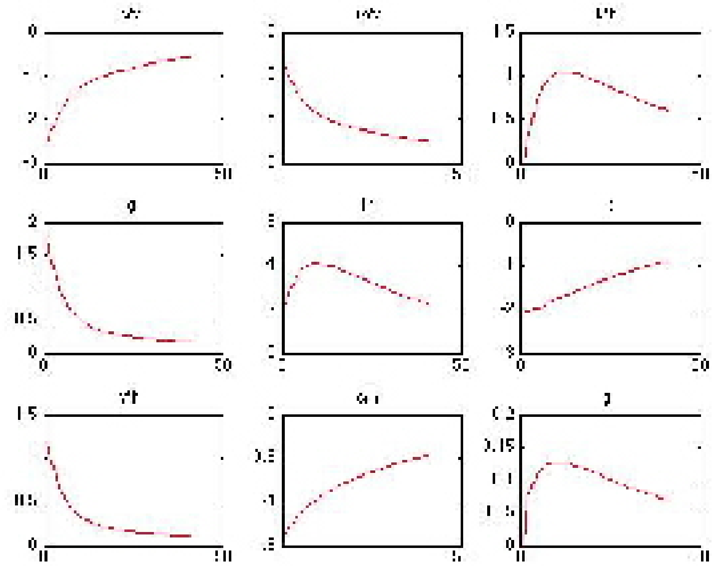
Our calibration uses a simulated annealing search algorithm that generalizes Jermann's (1998) methodology, whereby we also create convergence between the computer-chosen calibration parameters and the estimated parameters of the shocks that are identified using the Ingram and Kockerlakota (1997) and Benk et al. (2005) methodology of shock construction for the designated time period.

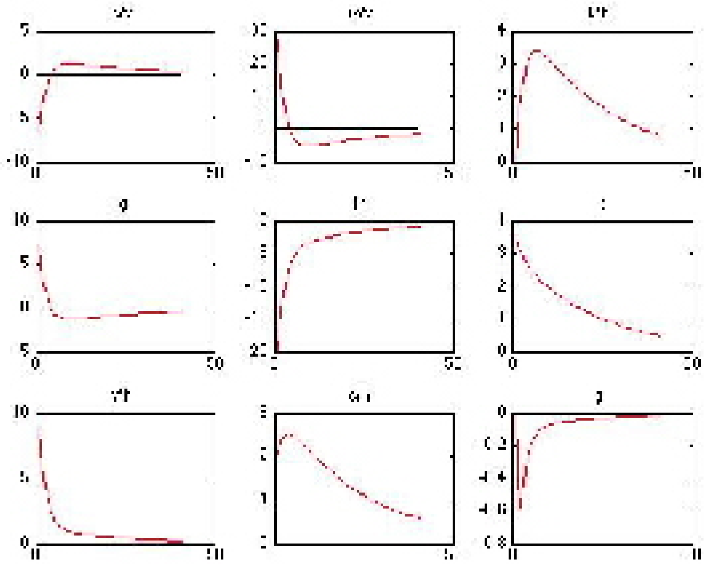
LR Values of Variable	Steady State Values		
	Model 1	Model 2	Target Values
Primary Target			
g	0.0035	0.0035	0.0035
u	N/A	0.78	0.78
c/y	0.8112	0.8	0.8
ik/y	0.188	0.2	0.2
lg	0.359	0.284	0.3
x	0.48	0.52	0.48
Secondary Targets			
Inflation	0.0067	0.004	0.005
R	0.0539	0.038	
lh	0.0266	0.196	
ik/h	0.0781	0.144	
y/h	0.4139	0.732	
c/h	0.3358	0.588	
k/h	2.7417	5.060	

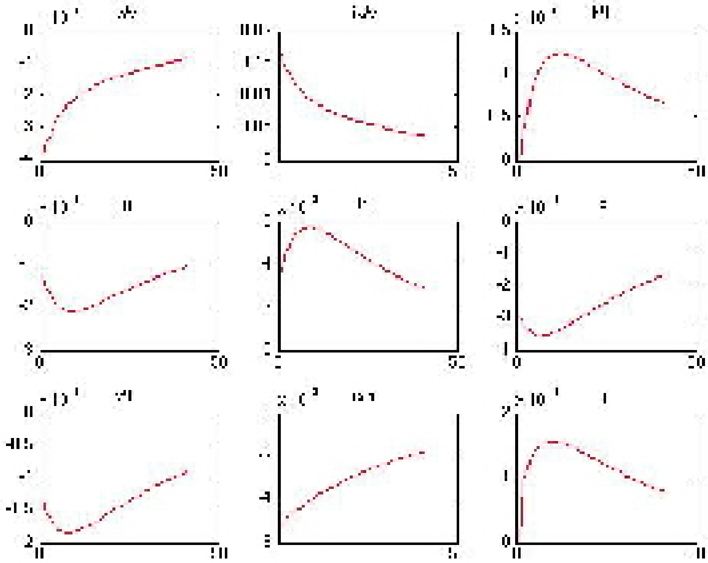
3.2. Impulse Responses

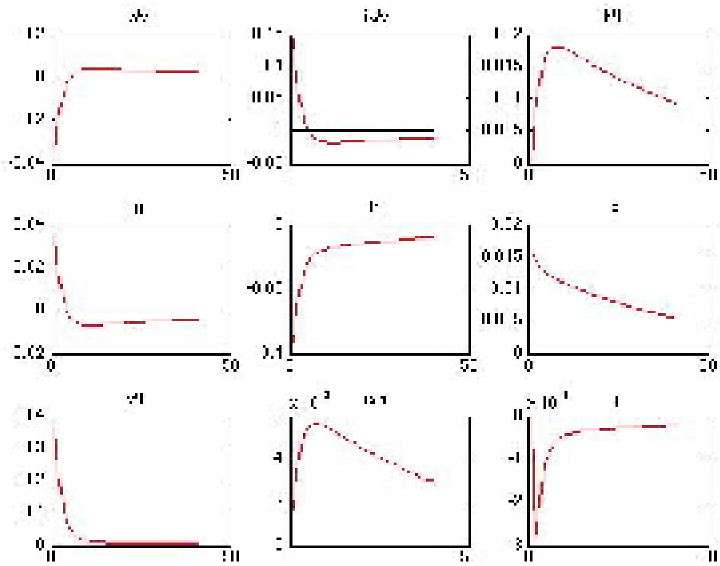
There are three shocks in the model of Section 2. One is the real productivity shock of the goods producing sector, another is the real productivity shock of the human investment sector, and the third is the money supply growth rate shock. These shocks are assumed to be highly correlated, even though not completely as in Benhabib et al. (2005) to have an aggregate real shock. Both of these shocks induce a negative relationship between GDP growth and inflation for all model variants.

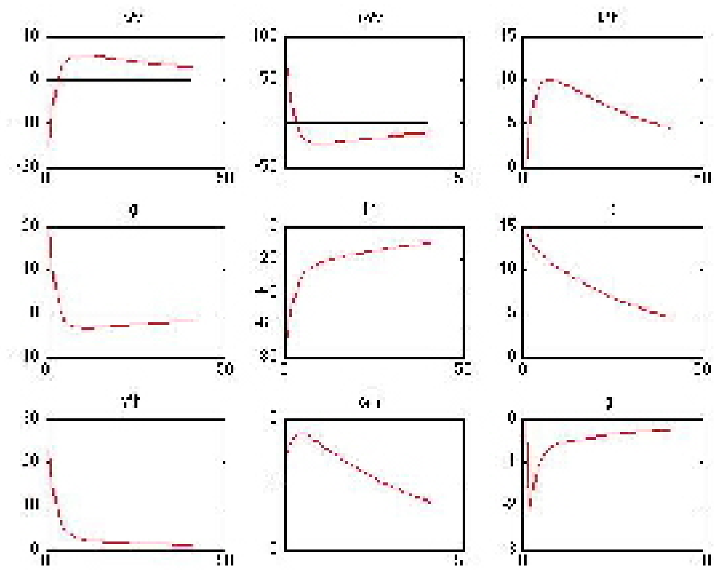












3.3. Comparison of Simulated Moments to Data Moments

		6_32 Business Cycle	32_200 Low Frequency	2_200 Medium Term
Corr(c,y)				
	data	0.895	0.978	0.962
	Model1	-0.601	-0.531	-0.404
	Model2	0.845	0.845	0.782
Corr(ik,y)				
	data	0.943	0.810	0.815
	Model1	0.990	0.907	0.950
	Model2	0.985	0.862	0.943
Corr(lg,y)				
	data	0.730	0.597	0.602
	Model1	0.890	0.851	0.705
	Model2	0.898	0.831	0.623
Corr(lh,y)				
	data	N/A	N/A	N/A
	Model1	-0.911	-0.672	-0.655
	Model2	-0.674	-0.492	-0.425
Corr(u,y)				
	data	0.791	0.363	0.440
	Model1	N/A	N/A	N/A
	Model2	0.799	0.265	0.410
Corr(c,ik)				
	data	0.846	0.721	0.693
	Model1	-0.686	-0.839	-0.669
	Model2	0.740	0.458	0.530
Corr(lg,c)				
	data	0.764	0.616	0.619
	Model1	-0.927	-0.981	-0.914
	Model2	0.831	0.421	0.438

		6_32	32_200	2_200
		Business Cycle	Low Frequency	Medium Term
Corr(gy,P)				
	data	-0.500	-0.755	-0.353
	Model1	-0.533	-0.546	-0.257
	Model2	-0.671	-0.833	-0.409
Corr(y,P)				
	data	0.164	0.028	0.033
	Model1	0.324	0.398	0.269
	Model2	0.270	0.151	0.130
Corr(c,P)				
	data	0.152	-0.024	-0.009
	Model1	-0.856	-0.605	-0.613
	Model2	0.680	0.569	0.553
Corr(ik,P)				
	data	0.224	0.158	0.143
	Model1	0.414	0.557	0.427
	Model2	0.120	-0.288	-0.119

4. The Data and the Econometric Specification

For the estimation we use annual data for the period 1980 - 2010. Most of the data comes from the World Bank Database. For the exact data sources please see Appendix 1. The following countries are included: Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, Slovak Republic, Spain, United Kingdom, United States.

As we need very specific variables, like financial sector wages and money base, data availability seems to be problematic at this stage. In Table 1. below we show the availability of all variables that we need. If we included all explanatory variables that we intended to use in our econometric model, the number of data points for which we had non-missing data would go down to as low as 102. Furthermore, our econometric model specification controls for cross-country dependence and slope heterogeneity (to be specified in the next section), therefore we need to have

Table 3		Volatilities		
		6_32	32_200	2_200
		Business Cycle	Low Frequency	Medium Term
g(Y)				
	data	0.0065	0.0047	0.0106
	Model 1	0.0056	0.0010	0.0114
	Model 2	0.0088	0.0022	0.0187
Y				
	data	0.0168	0.0458	0.0490
	Model 1	0.0120	0.0080	0.0159
	Model 2	0.0160	0.0297	0.0354
C				
	data	0.0099	0.0374	0.0388
	Model 1	0.0015	0.0049	0.0051
	Model 2	0.0056	0.0254	0.0261
Ik				
	data	0.0545	0.0885	0.1063
	Model 1	0.0657	0.0553	0.0918
	Model 2	0.0625	0.0722	0.1043
Ig				
	data	0.0050	0.0222	0.0228
	Model 1	0.0534	0.0048	0.0767
	Model 2	0.0131	0.0102	0.0190
U				
	data	0.0262	0.0322	0.0429
	Model 1	N/A	N/A	N/A
	Model 2	0.0012	0.0033	0.0036
Ih				
	data	N/A	N/A	N/A
	Model 1	0.0258	0.0210	0.0357
	Model 2	0.0405	0.0566	0.0745
Inflation				
	data	0.0043	0.0040	0.0072
	Model 1	0.3229	0.6138	0.7059
	Model 2	1.1712	2.9388	3.2184
m				
	data	0.0275	0.1068	0.1110
	Model 1	0.0014	0.0046	0.0048
	Model 2	0.0062	0.0255	0.0265

at least as many observations (years) per country as the number of explanatory variables plus the number of additional parameters. The maximum number of explanatory variables we can handle depends on the actual choice of variables.

Table 4.1: Summary Statistics

	N	mean	sd
GDP PP growth	279	1.018	0.023
Invetsments	279	21.153	3.282
Inflation rate	279	3.871	4.152
Tax rate	279	11.672	3.350
Capacity utilization rate	279	80.725	4.435
Education level	279	9.155	1.737
GDP PP relative to US GDP	279	1.916	1.004
Employment rate	279	63.953	7.777

4.1. Real Growth rate Equation

The endogenous variable is the real per-capita output (GDP) growth rate (t/t-1), the growth rate of nominal GDP divided by the GDP deflator; denoted for country j at time t as g_{jt}.

Annual or quarterly tax rate series may be the most constraining variables in terms of the size of the data set.

The righthandside variables, some of which may be have endogeneity with the Real output growth rate, and so need instrumental variables that pass the Sargan test, are as follows:

1. CPI inflation rate: π_{jt}

Calculated from 2005=100 Consumer price indices (CPI) as $\pi_{jt} = (\frac{CPI_{t+1}}{CPIt} - 1) * 100$ [2]

2. Corporate income tax rate: τ^k_{jt}

[2]Source: World Bank Databank

Total corporate income tax rate measures the amount of taxes and mandatory contributions payable by businesses after accounting for allowable deductions and exemptions as a share of commercial profits. Taxes withheld (such as personal income tax) or collected and remitted to tax authorities (such as value added taxes, sales taxes or goods and service taxes) are excluded. [3]

3. Personal income tax rate: τ^l_{jt}

The average income tax rate of a single person with no children earning 100% of the average wage. [4]

4. Capacity utilization rate: u_{jt}

This is a business survey-based variable. Mehodology may differ by countries. The usual survey question is the following: "At what capacity is your company currently operating (as a percentage of full capacity)?". The answer is a percentage. Take for manufacturing only. [5]

5. A measure of education level: h_{jt}

The average years of education completed among people over age 15. Available for every fifth years only, for years in between we used the latest available data. [6]

6. Similarly, the ratio of per capita income in the US to per capital income in the other countries, as a regional convergence variable: $\frac{y_t^{US}}{y_{jt}}$

7. Investment divided by GDP: the investment ratio: i_{jt}. [7]

Gross fixed capital formation (formerly gross domestic fixed investment); it includes land improvements (fences, ditches, drains, and so on); plant, machinery, and equipment purchases; the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings. [8]

8. Money stock (M2) divided by Nominal GDP (this is inverse velocity):

[3] Source: World Bank Databank

[4] Source: World Bank Databank, Taxing Wages. Available for 8 family categories.

[5] Source: OECD Business Tendency Survey, Fed

[6] Source: World Bank Databank, Education Statistics

[7] Source: World Bank Databank

[8] Source: World Bank Databank

$m_{jt}/y_{jt} \equiv \frac{1}{v_{jt}}$.
Money and quasi money (M2) comprise the sum of currency outside banks, demand deposits other than those of the central government, and the time, savings, and foreign currency deposits of resident sectors other than the central government. t corresponds to lines 34 and 35 in the International MoInetary Fund's (IMF) International Financial Statistics (IFS). [9]

9. Variance of the CPI inflation rate, as an alternative to the CPI inflation rate, since some of the literature emphasizes this over the inflation rate itself: σ^{π}_{jt}. Yearly variance of monthly inflation rates. [10]

10. Government Spending divided by GDP, or the share of government spending in GDP: $\frac{g_{jt}}{y_{jt}}$
General government final consumption expenditure (formerly general government consumption); it includes all government current expenditures for purchases of goods and services (including compensation of employees). It also includes most expenditures on national defense and security, but excludes government military expenditures that are part of government capital formation. [11]

The role of financial development has been emphasized by Ross Levine as [finally] not being important in terms of level, but in the innovation in that sector.

11. Private Credit divided by GDP: the traditional level of financial development, in Levine et al. (2000): $\frac{q_{jt}}{y_{jt}}$
Domestic credit to private sector refers to financial resources provided to the private sector, such as through loans, purchases of nonequity securities, and trade credits and other accounts receivable, that establish a claim for repayment. For some countries these claims include credit to public enterprises. [12]

12. Productivity in the banking sector; real wage as a proxy for productivity: w^q_{jt}
Total Labor costs (compensation of employees) in Financial intermediation (C65T67)

[9] Source: World Bank Databank
[10] Source: OECD.Stat
[11] Source: World Bank national accounts data, and OECD National Accounts data files.
[12] Source: World Bank Databank

on PPP divided by employment in pax. [13]

13. Labor force participation rate, and alternatively: l_{jt}

Labor force participation rate is the proportion of the population ages 15 and older that is economically active. [14]

14 Employment rate: e_{jt}

Employment to population ratio; the proportion of a country's population aged 15 and older that is employed. [15]

4.2. The Model Specification

The baseline specification derived from the theoretical model is

$$\begin{aligned} g_{jt} &= \beta_0 + \beta_1 \pi_{jt} + \beta_{1b(alt)} \sigma_{jt}^{\pi} + \beta_2 \left(\tau_{jt}^k + \tau_{jt}^l \right) + \beta_{2b(alt),3} \frac{g_{jt}}{y_{jt}} + \beta_4 u_{jt} \\ &\quad + \beta_5 h_{jt} l_{jt} + \beta_6 \frac{y_t^{US}}{y_{jt}} + \beta_7 \frac{1}{v_{jt}} + \beta_{7b(alt),8a} \frac{q_{jt}}{y_{jt}} + \beta_{7c,8b(alt),9} w_{jt}^q \\ &\quad + \beta_{10} e_{jt} + \varepsilon_{jt} \end{aligned}$$

where the disturbance term ε_{jt} is going to be defined below.

When estimating this model we have to take into consideration that it is estimated on a unbalanced moderate-T, moderate-N panel database.[16] Taking into account the literature of panel time series estimation (e.g., Eberhardt and Teal (2010, 2011) and Moscone and Tosetti (2010)) we assume that

- the macroeconomic variables may show cross-sectional dependence, and
- the effects of the explanatory variables may differ by countries.

To handle these problems we extend the baseline specification such that

- country-specific fixed effects and slope parameters are included in order to capture time-invariant cross-country heterogeneity,

[13] OECD.Stat, STAN Database for structural analysis, IMF WEO Database

[14] Source: World Bank Databank

[15] Source: World Bank Databank

[16] The sample consists of N=12 countries and T=14-29 year/countries.

- country-specific time effects, interpreted as observed common factors are also included to capture the heterogeneity of business cycle effect across countries, and
- time-variant unobserved common factors with country-specific factor loadings are also introduced to capture any unobserved systematic processes that may be driving our observable explanatory variables. As these are non-observed, they may represent the combination of a large number of factors.

Furthermore, in Model 1 (see later on) we also assume that the same unobservable common factors may be driving ϵ_{jt}, the disturbance terms as well. (Coakley, Fuertes and Smith (2006), see more details in the next section).

These extensions to the baseline empirical model result in a system of three equations as follows:

$$g_{jt} = \sum_{i=1}^{k} \beta_{jk} x_{kjt} + \varepsilon_{jt} \tag{4.1}$$

$$x_{kjt} = \alpha 1_j + \lambda_j f_t + \gamma_j t + u_{kjt} \tag{4.2}$$

$$\varepsilon_{jt} = \alpha 2_j + \lambda_j f_t + \nu_{jt} \tag{4.3}$$

where

- j stands for country and t for year, just as before
- β_{jk} stands for country-specific slope parameters of the observed explanatory variable k
- x_{kjt} stands for the explanatory variable k

- $\alpha 1_j$ and $\alpha 2_j$ are country-specific fixed effects
- λ_j are country-specific factor loadings on unobserved time-variant factor f_t
- γ_j are country-specific factor loadings on observed time effects
- u_{kjt} and ν_{jt} are white noise disturbance terms.

The estimation is carried out using the Pesaran (2006) Common Correlated Effects Mean Group estimator (CCEMG).[17] The main advantage of this estimation strategy is that it allows to estimate this general multifactor error model while the number of unobserved common factors does not need to be estimated. (Pesaran, 2006). The CCEMG filters the regressors by means of cross-section averages 'such that asymptotically as the cross-section dimension (N) tends to infinity, the differential effects of unobserved common factors are eliminated'. The estimation procedure is computed by ordinary least squares (OLS) applied to N country-specific auxiliary regressions where the observed regressors are augmented with cross-sectional averages of the dependent variable (here, average GDP growth rate) and the country-specific regressors. The country-specific factor loadings (λ_j) of the unobserved common factor (f_t) are approximated by the linear combination of the cross section averages of the dependent variable and the country-specific regressors.

5. Econometric Results

Overall, three versions of the above system were estimated:

- Model 1: λ_j country-specific factor loadings on the unobserved time-variant common factor f_t are assumed to be zero, i.e., only the observed time-variant common factors t are included along the observed explanatory variables in the model:

[17] The estimation is in fact done using the XTMG Stata module by Eberhardt (2011).

$$g_{jt} = \alpha_j + \sum_{i=1}^{k} \beta_{jk} x_{kjt} + \gamma_j t + \nu_{jt} \quad (5.1)$$

and it is estimated using the Pesaran and Smith (1995) mean group (MG) estimator.

- Model 2: γ_j country-specific factor loadings on observed time effect t are assumed to be zero, so only the unobserved time-variant common factors f_t are included along the observed explanatory variables in the model the following way:

$$g_{jt} = \sum_{i=1}^{k} \beta_{jk} x_{kjt} + \varepsilon_{jt} \quad (5.2)$$

$$x_{kjt} = \alpha 1_j + \lambda_j f_t + u_{kjt} \quad (5.3)$$

$$\varepsilon_{jt} = \alpha 2_j + \lambda_j f_t + \nu_{jt} \quad (5.4)$$

- Model 3: both the observed t and the unobserved f_t common factors are included along the observed explanatory variables in the model, so both λ_j and γ_j are estimated as it is specified in equations (4.1), (4.2) and (4.3).

Using a dataset of 12 OECD countries for the unbalaced period of 1981-2009 with 279 observation altogether, we get that the capacity utilization rate, along with the inflation rate and investments, is significantly explain real per capita GDP growth. (See Table 2.) Higher capacity utilization and investments ratio go together with higher growth, while the relation of inflation to development has a negative sign.

Table 5.1: Estimation results

	(1) Model_trend	(2) Model_cce	(3) Model_cce_trend
Investments	0.00187	0.00395*	0.00536*
	(1.41)	(2.19)	(2.53)
Inflation rate	-0.00305**	-0.00278**	-0.00382**
	(-3.12)	(-2.76)	(-3.17)
Tax rate	-0.00368*	-0.00618*	-0.00482
	(-2.25)	(-2.26)	(-1.40)
Capacity utilization	0.00526***	0.00313**	0.00248*
	(7.06)	(2.66)	(2.06)
Education	-0.0152	0.00562	0.00749
	(-1.30)	(0.91)	(1.05)
US GDPPP relative to country GDPPP	0.0520*	-0.0135	-0.0723**
	(2.36)	(-0.35)	(-2.94)
Employment rate	0.000793	0.0000975	-0.000457
	(0.36)	(0.07)	(-0.24)
Country-specific time trend	YES	NO	YES
Unobserved common factor	NO	YES	YES
Observations	279	279	279
Number of countries	12	12	12
Root Mean Squared Errors	0.0000563	0.0000136	0.0000123
chi2	280.0	27.72	74.50
p	1.10e-56	0.000247	1.81e-13

t statistics in parentheses

$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

6. Comments

If tax rate is included separately as individual and corporate (tau-ind + tau-corp), capacity utilization is significant in Model 1 only, while if tax rate is included as one variable (tau-kl), capacity utilization is significant in all three models. Also, if tau-kl is included alone, there are 279 observations in all three models.

Two questions may arise regarding our estimation strategy and our results. Firstly, why aren't all explanatory variables included in the model as listed above? This one is easy: because of data availability constraints. If financial sector productivity is included in addition of the variables in Table 2, due to the lack of available data, the number of observations collapses to 2/3. If private credit ratio is included, the sample with non-missing data gets to less than half of the original. Similarly, including government expenditure reduces sample size further. Considering that we estimate a heterogenous slope multifactor model and we need high degree of freedom, any reductions in sample size make the empirical model very sensitive. This is the reason why we stick to this reduced model for now, but it may be extended in future if more data sources emerged.

The second question is more difficult to answer: can we directly identify causal relationship in these models? Can we say that higher capacity utilization actually causes higher growth? To claim causality, we would have to conduct a thougth experiment, something like

1. lets have a random sample of countries;
2. again randomly, split the sample to two;
3. make sure the two groups have identical characteristics regarding growth possibilities;
4. induce one group to reach higher capacity utilization somehow while you leave the other group untouched;
5. make sure nothing else changes;

6. after a couple of years compare their growth rate.

As our data is neither from such an experiment nor from a similar natural-type experiment where changes in capacity utilization (or in an appropriate instrumental variable) are exogenous, as in most of the related literature, we can not directly identify causal effects here. However, this does not mean that we can not learn from this research. As for example Eberhardt and Teal (2011) points out, although the recent empirical development literature has gone to the direction of 'random experiments' and single-country 'growth diagnostics', we still can learn more from cross-country studies if we use appropriate methods, just as we did. In a large part of empirical growth literature even finding stable signs of correlation might not be trivial at all, especially if we control for cross-country dependence and observed and unobserved underlying factor structure.

7. Conclusion

Models of inflation and Lucas (1988) style endogenous growth have been used successfully to motivate estimation of the output growth rate, such as Gillman, Harris and Matyas (2004). Here we present this style of a $DSGE$ monetary endogenous growth model with a special case that is our Model 1; this is our Baseline model and it is similar to Gillman et al. except with a cash-only Clower constraint and without capacity utilization or taxes, as in Gomme (1993). Model 2 then includes a more general specification of the exchange constraint as in Stockman (1981), capacity utilization with convexity, and income taxes. Model 2 performs better across the business cycle, low frequency and Medium term spectra by most of the criteria as compared to the data.

Our econometric results are based on a Small-N panel estimation. They show significance of capacity utilization and taxes that appear also in our $DSGE$ Model 2. The negative effect of inflation on growth remains robust, as do the investment rate and the convergence variable, as in Gillman et al. (2004). The inclusion of capacity utilization and taxes merges together the standard growth theory with business cycle aspects to provide an explanation of what some call growth cycles.

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Appendix A. Equilibrium Conditions

Define the Lagrange multiplier of the consumer's budget constraint as λ_t, the exchange constraint's as μ_t, and that of the human capital accumulation's as ϕ_t. The household's first order conditions are the following,

$$c_t : \quad \frac{1}{c_t} = \lambda_t + \mu_t \tag{A-1}$$

$$l_{gt} : \quad \frac{\alpha}{x_t} = \lambda_t w_t h_t \tag{A-2}$$

$$l_{ht} : \quad \frac{\alpha}{x_t} = \phi_t \eta A_h z_t^h \left[\frac{l_{ht} h_t}{s_{ht} u_t k_t} \right]^{\eta - 1} h_t \tag{A-3}$$

$$\begin{aligned} u_t \quad : \quad & \frac{\chi}{(1-u_t)} = \lambda_t r_t s_{gt} k_t - \lambda_t \delta_k u_t^{\psi-1} k_t - \mu_t \delta_k u_t^{\psi-1} k_t \\ & + \phi_t (1-\eta) A_h z_t^h \left[\frac{l_{ht} h_t}{s_{ht} u_t k_t} \right]^{\eta} (s_{ht} k_t) \end{aligned} \tag{A-4}$$

$$s_{gt} : \quad \lambda_t r_t u_t k_t = \phi_t (1-\eta) A_h z_t^h \left[\frac{l_{ht} h_t}{s_{ht} u_t k_t} \right]^{\eta} (u_t k_t) \tag{A-5}$$

$$m_{t+1} : \quad \lambda_t (1 + \pi_{t+1}) = \beta E_t (\lambda_{t+1} + \mu_{t+1}) \tag{A-6}$$

$$b_{t+1} : \quad \lambda_t (1 + \pi_{t+1}) = \beta E_t \lambda_{t+1} (1 + R_{t+1}) \tag{A-7}$$

$$\begin{aligned} k_{t+1} : \quad \lambda_t + \mu_t = {} & \beta E_t \left[1 + r_{t+1} (s_{gt+1} u_{t+1}) - \frac{\delta_k}{\psi} u_{t+1}^{\psi} \right] + \beta E_t \mu_{t+1} \left[1 - \frac{\delta_k}{\psi} u_{t+1}^{\psi} \right] + \\ & + \beta E_t \phi_{t+1} (1-\eta) A_h z_{t+1}^h \left[\frac{l_{ht+1} h_{t+1}}{s_{ht+1} u_{t+1} k_{t+1}} \right]^{\eta} (s_{ht+1} u_{t+1}) \end{aligned} \tag{A-8}$$

$$h_{t+1}: \quad \phi_t = \beta E_t \phi_{t+1} \left[1 + \eta A_h z_{t+1}^h \left[\frac{l_{ht+1} h_{t+1}}{s_{ht+1} u_{t+1} k_{t+1}} \right]^{\eta-1} l_{ht+1} - \delta_h \right] + \beta E_t \lambda_{t+1} w_{t+1} l_{gt+1} \tag{A-9}$$

For the goods producer the associated standard first order conditions are

$$l_{gt}: w_t h_t = \gamma A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma-1} h_t \tag{A-10}$$

$$h_t: w_t l_{gt} = \gamma A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma-1} l_{gt} \tag{A-11}$$

$$u_t: r_t(s_{gt} k_t) = (1-\gamma) A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma} (s_{gt} k_t) \tag{A-12}$$

$$s_{gt}: r_t(u_t k_t) = (1-\gamma) A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma} (u_t k_t) \tag{A-13}$$

$$k_t: r_t(s_{gt} u_t) = (1-\gamma) A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma} (s_{gt} u_t) \tag{A-14}$$

The model can then be summarised by the following equations.

$$A_g z_t^g (l_{gt} h_t)^{\gamma} (s_{gt} u_t k_t)^{1-\gamma} = c_t + k_{t+1} - k_t + \frac{\delta_k}{\psi} u_t^{\psi} k_t \tag{A-15}$$

$$A_h z_t^h (l_{ht} h_t)^{\eta} ((1-s_{gt}) u_t k_t)^{1-\eta} = h_{t+1} - (1-\delta_h) h_t \tag{A-16}$$

$$p_{ht} = \left[\frac{A_g}{A_h} \right] \left[\frac{z_t^g}{z_t^h} \right] \left[\frac{\gamma}{\eta} \right]^{\eta} \left[\frac{1-\gamma}{1-\eta} \right]^{1-\eta} \left[\frac{s_{gt} u_t k_t}{l_{gt} h_t} \right]^{\eta-\gamma} \tag{A-17}$$

$$\frac{\alpha}{1 - l_{gt} - l_{ht}} \frac{c_t}{h_t} = \frac{\gamma A_g z_t^g \left[\frac{l_{gt} h_t}{s_{gt} u_t k_t} \right]^{\gamma-1}}{1 + R_t} \tag{A-18}$$

$$\frac{\chi}{(1-u_t)}\frac{c_t}{k_t} = \frac{(1-\gamma)A_g z_t^g \left[\frac{l_{gt}h_t}{s_{gt}u_t k_t}\right]^{\gamma}}{(1+R_t)} - \delta_k u_t^{\psi-1} \quad \text{(A-19)}$$

$$\frac{\gamma}{1-\gamma}\frac{l_{gt}h_t}{s_{gt}u_t k_t} = \frac{\eta}{1-\eta}\frac{l_{ht}h_t}{(1-s_{gt})u_t k_t} \quad \text{(A-20)}$$

$$(1+R_t) = E_t\left[\left[1 + \frac{(1-\gamma)A_g z_{t+1}^g \left[\frac{l_{gt+1}h_{t+1}}{s_{gt+1}u_{t+1}k_{t+1}}\right]^{\gamma} u_{t+1}}{1+R_{t+1}} - \frac{\delta_k}{\psi}u_{t+1}^{\psi}\right](1+\pi_{t+1})\right] \quad \text{(A-21)}$$

$$(1+\pi_{t+1})m_{t+1} = m_t(1+\sigma_t) \quad \text{(A-22)}$$

$$m_t = c_t + k_{t+1} - k_t + \frac{\delta_k}{\psi}u_t^{\psi}k_t \quad \text{(A-23)}$$

$$1 = \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)\left[1 + \frac{(1-\gamma)A_g z_{t+1}^g \left[\frac{l_{gt+1}h_{t+1}}{s_{gt+1}u_{t+1}k_{t+1}}\right]^{\gamma} u_{t+1}}{1+R_{t+1}} - \frac{\delta_k}{\psi}u_{t+1}^{\psi}\right]\right] \quad \text{(A-24)}$$

$$\begin{aligned} &1 \\ = \quad &\beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)\left(\frac{1+R_t}{1+R_{t+1}}\right)\left(\frac{p_{ht+1}}{p_{ht}}\right)\left[1 + (l_{gt+1}+l_{ht+1})\eta A_h z_{t+1}^h\left[\frac{l_{ht+1}h_{t+1}}{(1-s_{gt+1})u_{t+1}k_{t+1}}\right]^{\eta-1} - (A\delta\right.\right.\end{aligned}$$

Equation (A-15) is the goods market clearing condition; equation (A-16) is the human capital law of motion; equation (A-17) defines the relative price of human capital in units of consumption goods; equation (A-18) is the intra-temporal condition that governs the substitution between leisure and consumption; meanwhile, equation (A-19) is the second intra-temporal condition governing the substitution between managerial capacity and consumption. Equation (A-20) equates weighted factor intensities across sectors; (A-21) is the forward looking Fisher relationship;

equation (A-22) is the government's money supply rule; and (A-24) is the physical capital law of motion. Equations (A-24) and (A-25) are the inter-temporal capital efficiency conditions with respect to physical and human capital, where the capacity utilisation of physical capital is the equivalent of used managerial capacity, u_t, and the capacity utilisation of human capital is equivalent to total working time $(1 - x_t)$.

The set of 11 equations in (A-15) - (A-25) fully describes the model. Alltogether there are 11 equations in 11 unknowns $\{k_{t+1}, h_{t+1}, m_{t+1}, c_t, u_t, l_{gt}, l_{ht}, s_{gt}, \pi_t, p_{ht}, R_t\}$. Furthermore, the exogenous variables $\{z_t^g, z_t^h, \sigma_t\}$ are governed by the AR(1) processes defined in equations (3), (20), and (24).

Appendix B. The Steady State

Given that the uniqueness of the steady state is considered to be infeasible in such a models, here uniqueness of the steady state is demonstrated for a given calibration.[18] First, express the first order conditions in *Appendix A* in terms of the variables' long-run values. Variables without time subscripts represent the variables' long-run values. Then the first order conditions become

$$A_g \left[\frac{s_g uk}{l_g h}\right]^{\gamma} (us_g) = \frac{c}{k} + g + \frac{\delta_k}{\psi} u^{\psi} \tag{B-1}$$

$$A_h \left[\frac{(1-s_g)uk}{l_h h}\right]^{\eta-1} l_h = g + \delta_h \tag{B-2}$$

$$p_h = \left[\frac{A_g}{A_h}\right] \left[\frac{\gamma}{\eta}\right]^{\eta} \left[\frac{1-\gamma}{1-\eta}\right]^{1-\eta} \left[\frac{s_g uk}{l_g h}\right]^{\eta-\gamma} \tag{B-3}$$

$$\frac{\alpha}{1 - l_g - l_h} \frac{c}{h} = \frac{\gamma A_g \left[\frac{s_g uk}{l_g h}\right]^{1-\gamma}}{1+R} \tag{B-4}$$

[18] King and Rebelo, (1998)

$$\frac{\chi}{(1-u)}\frac{c}{k} = \frac{(1-\gamma)A_g\left[\frac{s_g uk}{l_g h}\right]^{-\gamma}}{(1+R)} - \delta_k u^{\psi-1} \tag{B-5}$$

$$\frac{1-\gamma}{\gamma}\frac{s_g uk}{l_g h} = \frac{1-\eta}{\eta}\frac{(1-s_g)uk}{l_h h} \tag{B-6}$$

$$(1+R) = \left[1 + \frac{(1-\gamma)A_g\left[\frac{s_g uk}{l_g h}\right]^{-\gamma} u}{1+R} - \frac{\delta_k}{\psi}u^{\psi}\right](1+\pi) \tag{B-7}$$

$$(1+\pi)(1+g) = (1+\bar{\sigma}) \tag{B-8}$$

$$m = c + \left[g + \frac{\delta_k}{\psi}u^{\psi}\right]k \tag{B-9}$$

$$1+g = \beta\left[1 + \frac{(1-\gamma)A_g\left[\frac{s_g uk}{l_g h}\right]^{-\gamma} u}{1+R} - \frac{\delta_k}{\psi}u^{\psi}\right] \tag{B-10}$$

$$1+g = \beta\left[1 + (l_g + l_h)\eta A_h\left[\frac{(1-s_g)uk}{l_h h}\right]^{1-\eta} - \delta_h\right] \tag{B-11}$$

where $1+g$ is the gross balanced growth rate of the economy. Define $f_g \equiv \frac{s_g uk}{l_g h}$ and $f_h \equiv \frac{(1-s_g)uk}{l_h h}$. Then the above system can be written in terms 11 unknowns $\{f_g, f_h, g, \frac{m}{k}, \frac{c}{k}, u, l_g, l_h, \pi, p_h, R\}$ as,

$$A_g f_g^{\gamma} u\left[\frac{l_g f_g}{l_g f_g + l_h f_h}\right] = \frac{c}{k} + g + \frac{\delta_k}{\psi}u^{\psi} \tag{B-12}$$

$$A_h f_h^{\eta-1} l_h = g + \delta_h \tag{B-13}$$

$$p_h = \left[\frac{A_g}{A_h}\right]\left[\frac{\gamma}{\eta}\right]^{\eta}\left[\frac{1-\gamma}{1-\eta}\right]^{1-\eta} f_g^{\eta-\gamma} \tag{B-14}$$

$$\frac{\alpha}{1-l_g-l_h}\frac{c}{k}\left[\frac{l_g f_g + l_h f_h}{u}\right] = \frac{\gamma A_g f_g^{1-\gamma}}{1+R} \tag{B-15}$$

$$\frac{\chi}{(1-u)}\frac{c}{k} = \frac{(1-\gamma)A_g f_g^{-\gamma}}{(1+R)} - \delta_k u^{\psi-1} \tag{B-16}$$

$$\frac{1-\gamma}{\gamma} f_g = \frac{1-\eta}{\eta} f_h \tag{B-17}$$

$$(1+R) = \left[1 + \frac{(1-\gamma)A_g f_g^{-\gamma} u}{1+R} - \frac{\delta_k}{\psi} u^{\psi}\right](1+\pi) \tag{B-18}$$

$$(1+\pi)(1+g) = (1+\bar{\sigma}) \tag{B-19}$$

$$\frac{m}{k} = \frac{c}{k} + \left[g + \frac{\delta_k}{\psi} u^{\psi}\right] \tag{B-20}$$

$$1+g = \beta\left[1 + \frac{(1-\gamma)A_g f_g^{-\gamma} u}{1+R} - \frac{\delta_k}{\psi} u^{\psi}\right] \tag{B-21}$$

$$1+g = \beta\left[1 + (l_g + l_h)\eta A_h f_h^{1-\eta} - \delta_h\right] \tag{B-22}$$

Given the exogenous information set of parameters $(\alpha, \chi, \beta, \gamma, \eta, \bar{\sigma}, \delta_k, \delta_h, \psi, A_g, A_h)$, the uniqueness of the solution to the system in (B-12) - (B-22) can be narrowed down to the uniqueness of variables g and u. In order to show this one can solve for $f_g, f_h, \frac{m}{k}, \frac{c}{k}, l_g, l_h, \pi, p_h, R$ in terms of g and u, which leaves a system of two implicit equations (B-15) and (B-16) in two unknowns g and u. First, one can solve for π using equation (B-19) as

$$\pi = \left[\frac{1+\bar{\sigma}}{1+g}\right] - 1. \tag{B-23}$$

From the Fisher relationship it follows that the nominal interest rate in the steady state is

$$R = \left[\frac{1+\bar{\sigma}}{1+g} - 1\right]\left[\frac{1+g}{\beta}\right]. \tag{B-24}$$

Then using the solution of R in terms of the BGP growth rate, g, one can express f_g in terms of g and u from equation (B-21):

$$f_g = \left[\frac{\left(\frac{1+g}{\beta} - 1 + \frac{\delta_k}{\psi}u^{\psi}\right)(1+R)}{(1-\gamma)A_g u}\right]^{\frac{1}{\gamma}}. \tag{B-25}$$

Then f_h directly follows from (B-17),

$$f_h = \frac{\eta}{\gamma}\frac{1-\gamma}{1-\eta}f_g. \tag{B-26}$$

Then one can express total labor time $(l_g + l_h)$ from equation (B-22):

$$B = (l_g + l_h) = \left[\frac{\frac{1+g}{\beta} - 1 + \delta_h}{\eta A_h f_h^{1-\eta}}\right]. \tag{B-27}$$

To express the time shares one can express l_h in terms of g from equation (B-13) and then use the solution for total labor time,

$$l_h = \left[\frac{g+\delta_h}{A_h}\right] f_h^{1-\eta}, \tag{B-28}$$

$$l_g = B - l_h. \tag{B-29}$$

Next by using equation (B-14) and the obtained expression for f_g it follows that the relative price of human capital in terms of g and u is,

$$p_h = \left[\frac{A_g}{A_h}\right]\left[\frac{\gamma}{\eta}\right]^{\eta}\left[\frac{1-\gamma}{1-\eta}\right]^{1-\eta} f_g^{\eta-\gamma} \tag{B-30}$$

Now one can obtain an expression in g and u for c/k from equation (B-12),

$$\frac{c}{k} = A_g f_g^{\gamma} u \left[\frac{l_g f_g}{l_g f_g + l_h f_h}\right] - g - \frac{\delta_k}{\psi}u^{\psi}, \tag{B-31}$$

from which m/k directly follows using equation (B-20).

Then after substituting (B-23) - (B-31) into equations (B-15) and (B-16) one obtains a highly nonlinear system of two equations in g and u: $\Phi(g,u)=0$. This system then can be solved numerically for the baseline calibration of parameters defined in Table 1.

Appendix C. Stochastic Normalisation and Solution Methodology

Such an endogenous growth model exhibits non-stationary features. Endogenous variables $\{k_{t+1}, h_{t+1}, m_t, c_t\}$ grow with a common rate along the BGP. In order to be able to solve the model in (A-15) - (A-25) one has to rewrite the system of equations in by using the following new stationary variables $g_{ht+1} \equiv \frac{h_{t+1}}{h_t}$; $\tilde{k}_t \equiv \frac{k_t}{h_t}$; $\tilde{m}_t \equiv \frac{m_t}{h_t}$; $\tilde{y}_t \equiv \frac{y_t}{h_t}$; $\tilde{i}_{kt} \equiv \frac{i_{kt}}{h_t}$; and $\tilde{c}_t \equiv \frac{c_t}{h_t}$.

$$A_g z_t^g l_{gt}^{\gamma}(s_{gt}u_t\tilde{k}_t)^{1-\gamma} = \tilde{c}_t + \tilde{k}_{t+1}g_{ht+1} - \tilde{k}_t + \frac{\delta_k}{\psi}u_t^{\psi}\tilde{k}_t \tag{C-1}$$

$$A_h z_t^h l_{ht}^{\eta}((1-s_{gt})u_t\tilde{k}_t)^{1-\eta} = g_{ht+1} - (1-\delta_h) \tag{C-2}$$

$$p_{ht} = \left[\frac{A_g}{A_h}\right]\left[\frac{z_t^g}{z_t^h}\right]\left[\frac{\gamma}{\eta}\right]^{\eta}\left[\frac{1-\gamma}{1-\eta}\right]^{1-\eta}\left[\frac{s_{gt}u_t\tilde{k}_t}{l_{gt}}\right]^{\eta-\gamma} \tag{C-3}$$

$$\frac{\alpha}{1-l_{gt}-l_{ht}}\tilde{c}_t = \frac{\gamma A_g z_t^g\left[\frac{l_{gt}}{s_{gt}u_t\tilde{k}_t}\right]^{\gamma-1}}{1+R_t} \tag{C-4}$$

$$\frac{\chi}{(1-u_t)}\frac{\tilde{c}_t}{\tilde{k}_t} = \frac{(1-\gamma)A_g z_t^g\left[\frac{l_{gt}}{s_{gt}u_t\tilde{k}_t}\right]^{\gamma}}{(1+R_t)} - \delta_k u_t^{\psi-1} \tag{C-5}$$

$$\frac{\gamma}{1-\gamma}\frac{l_{gt}}{s_{gt}u_t\tilde{k}_t} = \frac{\eta}{1-\eta}\frac{l_{ht}}{(1-s_{gt})u_t\tilde{k}_t} \tag{C-6}$$

$$(1+R_t) = E_t\left[\left[1+\frac{(1-\gamma)A_g z^g_{t+1}\left[\frac{l_{gt+1}}{s_{gt+1}u_{t+1}\tilde{k}_{t+1}}\right]^{\gamma}u_{t+1}}{1+R_{t+1}}-\frac{\delta_k}{\psi}u^{\psi}_{t+1}\right](1+\pi_{t+1})\right] \tag{C-7}$$

$$(1+\pi_{t+1})\tilde{m}_{t+1}g_{ht+1} = \tilde{m}_t(1+\sigma_t) \tag{C-8}$$

$$\tilde{m}_t = \tilde{c}_t + \tilde{k}_{t+1}g_{ht+1} - \tilde{k}_t + \frac{\delta_k}{\psi}u^{\psi}_t\tilde{k}_t \tag{C-9}$$

$$1 = \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)\left[1+\frac{(1-\gamma)A_g z^g_{t+1}\left[\frac{l_{gt+1}}{s_{gt+1}u_{t+1}\tilde{k}_{t+1}}\right]^{\gamma}u_{t+1}}{1+R_{t+1}}-\frac{\delta_k}{\psi}u^{\psi}_{t+1}\right]\right] \tag{C-10}$$

$$1 = \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)\left(\frac{1+R_t}{1+R_{t+1}}\right)\left(\frac{p_{ht+1}}{p_{ht}}\right)\left[1+(l_{gt+1}+l_{ht+1})\eta A_h z^h_{t+1}\left[\frac{l_{ht+1}}{(1-s_{gt+1})u_{t+1}\tilde{k}_{t+1}}\right]^{\eta-1} - \delta \right.\right. \tag{C-11}$$

The next step, given a steady state solution to the above model, is to log-linearise the above first order conditions using a first-order Taylor expansion, which characterise the equilibrium of the stationary model. This way all equations in (C-1) to (C-11) become approximately linear in the log-deviations from the steady state of the underlying variables.

The log-linearised version of the system in (C-1) to (C-11) is the following:

$$\begin{aligned} 0 \;\approx\;& \hat{z}^g_t + \gamma\hat{l}_{gt} + (1-\gamma)\hat{s}_{gt} + \\ & \left[(1-\gamma) - \delta_k u^{\psi}k\right]\hat{u}_t + \left[(1-\gamma) + \frac{k - \frac{\delta_k}{\psi}u^{\psi}k}{y}\right]\hat{k}_{t-1} \\ & -\frac{c}{y}\hat{c}_t - \left[\frac{kg_h}{y}\right]\hat{k}_t - \left[\frac{kg_h}{y}\right]\hat{g}_{ht}, \end{aligned} \tag{C-12}$$

where $y = \left[A_g z^g l_g^{\gamma}(s_g uk)^{1-\gamma}\right]$.

$$0 \approx \hat{z}_t^h + \eta\hat{l}_{ht} - (1-\eta)\left[\frac{s_g}{1-s_g}\right]\hat{s}_{gt} + (1-\eta)\hat{u}_t + (1-\eta)\hat{k}_{t-1} - \frac{g_h}{i_h}g_{ht}, \qquad \text{(C-13)}$$

where $i_h = \left[A_h z^h l_h^{\eta}((1-s_g)uk)^{1-\eta}\right]$.

$$0 \approx -\hat{p}_{ht} + \hat{z}_t^g - \hat{z}_t^h + (\eta-\gamma)\hat{s}_{gt} + (\eta-\gamma)\hat{u}_t + (\eta-\gamma)\hat{k}_{t-1} + (\gamma-\eta)\hat{l}_{gt}, \qquad \text{(C-14)}$$

$$\begin{aligned} 0 \;\approx\; & \left[\frac{l_g}{1-l_g-l_h} + (\gamma-1)\right]\hat{l}_{gt} + \left[\frac{l_h}{1-l_g-l_h}\right]\hat{l}_{ht} - \hat{c}_t \\ & - \left[\frac{R}{1+R}\right]\hat{R}_t + \hat{z}_t^g + (1-\gamma)\hat{s}_{gt} + (1-\gamma)\hat{u}_t + (1-\gamma)\hat{k}_{t-1}, \end{aligned} \qquad \text{(C-15)}$$

$$\begin{aligned} 0 \;\approx\; & \left[\frac{u}{1-u} - \frac{\delta_k(\psi-1)u^{\psi-1}}{\frac{r}{1+R} - \delta_k u^{\psi-1}}\right]\hat{u}_t + \left[\frac{\frac{r}{1+R}}{\frac{r}{1+R} - \delta_k u^{\psi-1}}\right]\hat{r}_t \\ & + \left[\frac{\frac{rR}{(1+R)^2}}{\frac{r}{1+R} - \delta_k u^{\psi-1}}\right]\hat{R}_t - \hat{c}_t + \hat{k}_{t-1}, \end{aligned} \qquad \text{(C-16)}$$

where $\hat{r}_t = \hat{z}_t^g + \gamma\hat{l}_{gt} - \gamma\hat{s}_{gt} - \gamma\hat{u}_t - \gamma\hat{k}_{t-1}$.

$$0 \approx \hat{l}_{gt} - \hat{l}_{ht} + \left[\frac{s_g}{1-s_g} - 1\right]\hat{s}_{gt}, \qquad \text{(C-17)}$$

$$\begin{aligned} 0 \;\approx\; & \left[\frac{\pi}{1+\pi}\right]\hat{\pi}_{t+1} - \left[\frac{R}{1+R}\right]\hat{R}_t + \left[\frac{\frac{ru}{1+R}}{1 + \frac{ru}{1+R} - \frac{\delta_k}{\psi}u^{\psi}}\right]\hat{r}_{t+1} \\ & + \left[\frac{\frac{ruR}{(1+R)^2}}{1 + \frac{ru}{1+R} - \frac{\delta_k}{\psi}u^{\psi}}\right]\hat{R}_{t+1} + \left[\frac{\frac{ru}{1+R} - \delta_k u^{\psi}}{1 + \frac{ru}{1+R} - \frac{\delta_k}{\psi}u^{\psi}}\right]\hat{u}_{t+1}, \end{aligned} \qquad \text{(C-18)}$$

$$0 \approx \left[\frac{\pi}{1+\pi}\right]\hat{\pi}_{t+1} + \hat{m}_{t+1} + \hat{g}_{ht+1} - \hat{m}_t - \left[\frac{\sigma}{1+\sigma}\right]\hat{\sigma}_t, \qquad \text{(C-19)}$$

$$0 \approx -\hat{m}_t + \left[\frac{c}{m}\right]\hat{c}_t + \left[\frac{kg_h}{m}\right]\hat{k}_t + \left[\frac{kg_h}{m}\right]\hat{g}_{ht} + \left[\frac{\frac{\delta_k}{\psi}u^{\psi}k - k}{m}\right]\hat{k}_{t-1} + \left[\frac{\delta_k u^{\psi}}{m}\right]\hat{u}_t, \quad \text{(C-20)}$$

$$\begin{aligned} &0 \\ \approx \quad & E_t \left[\begin{array}{c} \hat{c}_t - \hat{c}_{t+1} - \hat{g}_{t+1} + \left[\frac{\frac{ru}{1+R}}{1+\frac{ru}{1+R}-\frac{\delta_k}{\psi}u^{\psi}}\right]\hat{r}_{t+1} \\ + \left[\frac{\frac{ruR}{(1+R)^2}}{1+\frac{ru}{1+R}-\frac{\delta_k}{\psi}u^{\psi}}\right]\hat{R}_{t+1} + \left[\frac{\frac{ru}{1+R}-\delta_k u^{\psi}}{1+\frac{ru}{1+R}-\frac{\delta_k}{\psi}u^{\psi}}\right]\hat{u}_{t+1} \end{array} \right], \end{aligned} \quad \text{(C-21)}$$

$$\begin{aligned} 0 \approx & \\ & E_t \left[\begin{array}{c} \hat{c}_t - \hat{c}_{t+1} - \hat{g}_{t+1} + \left[\frac{R}{1+R}\right]\hat{R}_t - \left[\frac{R}{1+R}\right]\hat{R}_{t+1} + \hat{p}_{ht+1} - \hat{p}_{ht} \\ + \left[\frac{xr^h}{1+(1-x)r^h-\delta_h}\right]\hat{x}_{t+1} + \left[\frac{(1-x)r^h}{1+(1-x)r^h-\delta_h}\right]\hat{r}^h_{t+1} \end{array} \right], \end{aligned} \quad \text{(C-22)}$$

where $\hat{x}_t \approx -(l_g/x)\hat{l}_{gt} - (l_h/x)\hat{l}_{ht}$, and $\hat{r}^h_t = \hat{z}^h_t + (1-\eta)\hat{l}_{ht} + (\eta-1)\hat{s}_{ht} + (\eta - 1)\hat{u}_t + (\eta-1)\hat{k}_{t-1}$, are the log-linear approximations of leisure, x_t, and the return on human capital r^h_t, respectively. Note that variables with a '$\hat{\cdot}$' represent log-linear approximations of variables from their respective steady states, meanwhile, variables without a time subscript denote steady state values of variables.

After obtaining the log-linear system of equations in equations (C-12) - (C-22), the method proposed by Uhlig (1999) is applied to solve the model and obtain the recursive policy functions. Uhlig's method based on the method of undetermined coefficients following King, Plosser, and Rebelo (1988a,b) among many is chosen as it is relatively simple to implement. In order to be able to apply this solution method one has to rewrite the log-linear version of the above first order conditions in the following matrix form:

$$Ax_t + Bx_{t-1} + Cy_t + Dz_t = 0, \quad \text{(C-23)}$$

$$E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_tLz_{t+1} + Mz_t] = 0, \quad \text{(C-24)}$$

$$z_{t+1} = Nz_t + \epsilon_{t+1}, \quad \text{(C-25)}$$

where $E_t(\epsilon_{t+1}) = 0$; the vector x_t (size $mx1$) contains the endogenous state variables; y_t (size $nx1$) is the vector of all other endogenous variables; meanwhile, z_t (size $kx1$) is the vector of exogenous stochastic variables. Is is assumed that the coefficient matrix C is of size lxn, where $l \geq n$ and of rank n. l is the number of deterministic equations [i.e. the number of equations in (C-23)], F is a coefficient matrix of size $(m + n - l)xm$, and N has only stable eigenvalues. In the underlying baseline model x_t contains the log-linear versions of $\hat{k}_t$ and $\hat{g}_{ht}$. There are three exogenous variables in z_t, namely, $\hat{z}_t^g$, $\hat{z}_t^h$, and σ_t. All other nine endogenous variables are in y_t, these are $\hat{m}_t$, $\hat{c}_t$, $\hat{u}_t$, $\hat{l}_{gt}$, $\hat{l}_{ht}$, $\hat{s}_{gt}$, $\hat{\pi}_t$, $\hat{p}_{ht}$, $\hat{R}_t$.[19]

The log-linear solution method by Uhlig (1999) is seeking to find a recursive equilibrium law of motion of the following form:

$$x_t = Px_{t-1} + Qz_t \quad \text{(C-26)}$$

$$y_t = Rx_{t-1} + Sz_t \quad \text{(C-27)}$$

Therefore, the underlying solution method is looking P, Q, R, and S so that the equilibrium described by these rules is stable in nature. In the case of the baseline model [and all extensions], $l = n$, then [20]

(i) P must satisfy the following quadratic matrix equation:

$$(F - JC^{-1}A)P^2 - (JC^{-1}B - G + KC^{-1}A)P - KC^{-1}B + H = 0 \quad \text{(C-28)}$$

(ii) R is given by

[19] The growth rate of human capital $g_{ht} \equiv h_{t+1}/h_t$ is defined as a state variable in order to satisfy the requirement that $l \geq n$ condition imposed by the log-linear approximation. Since it is not a state variable in the proper sense it will vanish from the recursive policy functions.

[20] For more details see Corollary 1 in Uhlig (1999).

$$R = -C^{-1}(AP + B) \tag{C-29}$$

(iii) Q satisfies

$$\begin{aligned} & vec(Q) \\ = \;& (N'^{-1}A) + I_k \otimes (JR + FP + G - KC^{-1}A))^{-1} vec((JC^{-1}D - L)N + KC^{-1}D - M)), \end{aligned} \tag{C-30}$$

where $vec(.)$ denotes column wise vectorisation.

(iv) Lastly, S is given by

$$S = -C^{-1}(AQ + D). \tag{C-31}$$

In order to have a stationary recursive solution, the key is to pick up the solution for P, whose eigenvalues are both smaller than unity. Given P the solution to R, Q, and S directly follow.

Appendix D. DSGE Comparison Data Sources

Data used in the paper's $DSGE$ comparison to simulated moments is of quarterly frequency, for the 1971:2 until 2012:2 time period. The three sources include data constructed by Gomme and Rupert (2007) available at *http://clevelandfed.org/research/Models/rbc* data from the Federal Reserve Bank, St. Louis database and data from the Congressional Budget Office (CBO). Gross Domestic Product (y), Personal Consumption Expenditures (c), Gross Private Domestic Investment (i_k), and Average Weekly Labor Hours (l_g) are from the database of Gomme and Rupert (2007). The Consumer Price Index data used for the construction of the inflation series (π), the quarterly Factor Utilisation Rate for all industries (u), the 3-months T-bill rate (R), currency (M), and the effective corporate tax rate are from the Federal Reserve Bank, St. Louis database. Data for the effective personal income tax rate is obtained from the CBO. Note that the effective tax rate series are of annual frequency due to lack of availability of quarterly data, meanwhile, the series for personal income tax is from 1979 only. The effective tax rate series were used to calibrate the exogenous tax processes. The human and physical capital stock data are constructed as in Gomme and Ruppert (2007) and Barro's website, respectively.