Global Dependence and Productivity: A Robust Nonparametric World Frontier Analysis

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Abstract: Increasing globalization and interconnection among countries generates spatial and temporal dependence which will affect the production process of each country. Many studies have analyzed the effect of cross-sectional dependence by using restrictive parametric models. We use rather a flexible nonparametric two-step approach on conditional efficiencies to eliminate the dependence of production inputs/outputs on these common factors. By using a dataset of 44 countries over 1970-2007, we estimate the world frontier and explore the channels under which Foreign Direct Investment (FDI) and time affect the production process and its components: impact on the attainable production set (input-output space), and the impact on the distribution of efficiencies. We extend existing methodological tools - robust frontier in non parametric location-scale models - to examine these interrelationships. We emphasize the usefulness of "pre-whitened" inputs/outputs to obtain more reliable measure of productivity and efficiency to better investigate the driven forces behind the catching-up productivity process. Furthermore, since the influence of external factors has been eliminated, our approach mitigates the problem of endogeneity bias caused by reverse causality between the external factors as FDI and productivity.

JEL: C14, C13, C33, D24, O47.

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1 Introduction

Due to an increasing globalisation and interconnection among countries through history, geography and trade relations, technological interdependency generated by externalities is important in explaining conditional convergence process across countries. Total factor productivity has been recognized as the most important driver behind economic growth (Prescott 1998, Caselli 2005, Parente and Prescott 2005). The issue of cross section dependency or correlation has been widely discussed in the empirical panel data literature (Bai and Ng 2006, Pesaran 2006, Bai 2009, Kapetanios et al. 2011). The productivity analysis also recognizes an importance of investigating the spillover effects of the global shocks and business cycles. Mastromarco et al. (2013) among others, demonstrate that it is crucially important to take into account globalisation factors for an analysis of productivity and output growth. Due to a certain degree of cross-section dependence (CSD) introduced by unobserved (heterogeneous) time-specific factors the conventional estimators would be seriously biased. The literature deals with cross section dependence, attributable to economy-wide shocks that affect all units in the cross section but with different intensities, by assuming a multi-factor error process characterized by a finite number of unobserved common factors. According to this approach, the error term is a linear combination of a few common time-specific effects with heterogeneous factor loadings plus an idiosyncratic (individual-specific) error term.

Chudik et al. (2011) introduce the distinction between weak and strong cross section dependence. Specifically, a process is said to be cross sectionally weakly dependent at a given point in time, if its weighted average at that time converges to its expectation in quadratic mean, as the cross section dimension is increased without bounds. If this condition does not hold, then the process is said to be cross sectionally strongly dependent. The distinctive feature of strong correlation is that it is pervasive, in the sense that it remains common to all units however large the number of cross sectional units.[1]

Pesaran (2006) and Bai (2009) propose two alternative way to handle strong cross sectional dependence. Pesaran (2006) suggests a pooled common correlated estimator (PCCE) which approximates the linear combinations of the unobserved factors by cross section averages of the dependent and explanatory variables and then runs standard panel regressions augmented with the cross section averages. An advantage of this approach is that it yields consistent estimates even when the regressors are correlated with the factors, and the number of factors are unknown. Bai (2009) proposes a principal component (PC) interactive

[1]Spatial dependence typically entertained in the literature turns out to be weakly dependent in this framework.

maximum likelihood estimator where the unobserved factors are identified by principal components. More recently Pesaran and Tosetti (2011) have presented a panel model in which the errors are a combination of a multifactor structure and a spatial process, hence combining strong and weak CSD.

So far all of the studies analyzing effect of external common factors on productivity of countries have been in the stream of parametric modeling. However, the parametric approach suffers of misspecification problems when the data generating process is unknown, as usual in the applied studies, and the nonparametric methods often give the most reliable results. The purpose of this paper is to provide fully nonparametric location scale estimators of production frontiers and time variant technical efficiency in a dynamic framework which allows external and global (time specific) factors to affect technical efficiency.[2] Our model constitutes an attempt to introduce, in a simple way, cross sectional dependence and correlation into a fully nonparametric panel modelling framework.

There is a fundamental measurement problem for total factor productivity (TFP). The usual approach to estimate TFP is through growth accounting to explain output growth as the accumulation of factor inputs and the growth of TFP. However this approach has an important drawback since it does not consider non-competitive markets, increasing returns to scale and factor utilisation over the business cycle. More importantly, growth accounting interprets the TFP (Solow residual) as "technical change". The interpretation of the TFP as technical change is reasonable only if all countries are producing on their frontier. Beyond factor inputs, we could have additional determinants of output growth affecting the efficiency with which real inputs are transformed into output and thus directly affecting productivity. TFP comprises two mutually exclusive parts, technological change and efficiency change, and frontier model allows us to distinguish between the two. Moreover our frontier model enables us to see whether the effect of environmental/global variables on productivity occurs via technology change or efficiency. We can then quantify the impact of environmental/global factors on efficiency levels and make inferences about the contributions of these variables in affecting efficiency.

In a macroeconomics context, as the one used in this paper, where countries are producers of output (i.e., GDP) given inputs (e.g., capital, labor, and technology), inefficiency can be identified as the distance of the individual production from the frontier estimated by the maximum output of the reference country regarded as the empirical counterpart of an

[2] The efficiency frontier literature defines environmental or external factors, those variables which might affect the production process but which are not under the direct control of the production unit.

optimal boundary of the production set. Inefficiencies generally reflect a sluggish adoption of new technologies, and thus efficiency improvement will represent productivity catch-up via technology diffusion.

We propose a flexible non parametric two step approach to take into account the cross section dependence due to common factors attributable to global shocks. Following recent development in non parametric conditional frontier literature (Florens et al. 2014) we suggest a nonparametric location-scale frontier model linking production inputs and output to the global and environmental factors. In the first step we clean the dependence of inputs and outputs on global and other environmental factors. In the second step we estimate the world frontier and the efficiency using inputs and outputs whitened from the influence of global shocks and endogenous environmental factors. We also define a robust version of the frontier estimates, robust to extreme and outlying values. By eliminating influence of external factors our nonparametric estimator is also robust to the endogeneity bias caused by reverse causality between external factors as FDI (foreign direct investment) and productivity. Our approach deals with endogeneity by proposing an estimator of the boundary of the production set based on 'cleaned' output and inputs which are uncorrelated with global factors and FDI.

More fundamentally, we propose a robust method which simultaneously addresses the problem of model specification uncertainty, potential endogeneity and spatial dependence in the analysis of productivity. It also accounts for heteroskedasticity.

The paper aims to examine the productivity catching up process using 44 countries over the period 1970-2009 and to investigate the role of global factor as FDI and time in spurring technological catching up (efficiency) among countries. This data set has been used in our previous paper (Mastromarco and Simar 2014) where the focus is on the time dependence of conditional efficiency frontier. Here we propose a different method. The main advantage of the approach used here, is that it needs smoothing in the conditional variables in the center of the data cloud and not at the frontier where there are fewer observations, therefore is more robust than our previous method. Moreover it allows to consider cross sectional dependence which is very important in the analysis of productivity at cross country level and, somehow, also endogeneity due to reverse causation. We then obtain an efficiency measure which is a cleaned from time and other conditional effects and enables us to better evaluate the economic performance of the countries and a better ranking.

2 The Methodology

We apply Florens et al. (2014) methodology and consider a Data Generating Process (DGP) characterizing the production process in the presence of environmental factors and we extend their models to a dynamic framework to allow the introduction of the time dimension and cross sectional dependence (CSD). Consider a generic input vector $X \in \mathbb{R}_+^p$, a generic output vector $Y \in \mathbb{R}_+$ and we will denote by $Z \in \mathbb{R}^r$ the generic vector of environmental variables (FDI in our study). Since we are in a context of panel data, our sample will be denoted by (X_{it}, Y_{it}, Z_{it}), with $i = 1, \ldots, n$ being the firm index and $t = 1, \ldots, s$ the time index. To better investigate the influence of globalization factors (e.g., technological shocks and financial crises) on the economic performance of countries under analysis, we develop a method to envelop the effect of CSD on the production process. Hence, we assume that the production process is function of unobserved time-varying factors. As proposed by Pesaran (2006), Bai (2009) we will consider $F_t = (t, X_{\cdot t}, Y_{\cdot t})$ as proxy for the unobserved nonlinear and complex trending patterns associated with globalisation and the business-cycle.[3]

2.1 A short excursion in Frontier models

For unfamiliar readers, we can summarize the setup of frontier models as follows. The production process is a process generating pairs of inputs $X \in \mathbb{R}_+^p$ and outputs $Y \in \mathbb{R}_+$. We first define the unconditional (marginal) attainable set of feasible combinations of inputs and outputs as $\Psi = \{(x, y) \in \mathbb{R}_+^{p+1} | x \text{ can produce } y\}$. It can be characterized by $\Psi = \{(x, y) | H_{X,Y}(x, y) > 0\}$, where $H_{X,Y}(x, y) = \text{Prob}(X \leq x, Y \geq y)$. So Ψ is the support of the joint random variable (X, Y). For the univariate output case, the frontier function can be defined for an input vector x as

$$\tau(x) = \sup\{y | H_{XY}(x, y) > 0\} = \sup\{y | S_{Y|X}(y|x) > 0\}, \tag{1}$$

where the conditional survivor function is $S_{Y|X}(y|x) = \text{Prob}(Y \geq y | X \leq x)$. Sometime, researchers report also for a unit operating at the level (x, y) the Farrell-Debreu output efficiency score $\lambda(x, y) = \tau(x)/y = \sup\{\lambda | S_{Y|X}(\lambda y|x) > 0\} \geq 1$. An efficiency score equal to one, detects a unit on the efficient frontier.

When we want to condition the frontier analysis to some environmental factors (Z, F_t), as is our setup here, we have rather to define the attainable set $\Psi^{z,f_t} \subset \mathbb{R}_+^{p+1}$ as the support

[3] Here we use the standard notation where a dot in a subscript, means that we averaged over this index.

of the conditional probability (Cazals et al. 2002):

$$H_{X,Y|Z,F_t}(x,y|z,f_t) = \text{Prob}\left(X \le x, Y \ge y \,|\, Z = z, F_t = f_t\right). \tag{2}$$

Accordingly, and following Daraio and Simar (2005), when the output is univariate, the conditional frontier function at input x, facing conditions z and f_t (in particular at time t), is defined as[4]

$$\tau(x,z,f_t) = \sup\{y|S_{Y|X,Z,F_t}(y|x,z,f_t) > 0\}, \tag{3}$$

where $S_{Y|X,Z,F_t}(y|x,z,f_t) = \text{Prob}(Y \ge y|X \le x, Z = z, F_t = f_t)$ (note the difference in the conditioning for X, the inputs, and for Z and F_t, the environmental and global factors). Again we can report the Farrell-Debreu conditional efficiency scores as

$$\lambda\left(x,y|z,f_t\right) = \tau(x,z,f_t)/y = \sup\{\lambda|S_{Y|X,Z,F_t}(\lambda y|x,z,f_t) > 0\}. \tag{4}$$

Nonparametric estimators of the attainable sets can be obtained by plugging nonparametric estimators of the survivor functions in the definitions above. Plugging the empirical version of $S_{Y|X}$ in (1) provides the popular FDH (Free Disposal Hull) estimator of Ψ. A nonparametric estimator of the conditional survival function $S_{Y|X,Z,F_t}(y|x,z,f_t)$ could be obtained by using standard smoothing methods where a bandwidth h has to be determined for each component of (Z, F_t) (as e.g. in Badin et al., 2010). In summary, these nonparametric estimators are consistent with rate $n^{1/(p+1)}$ and Weibull limiting distribution for the unconditional FDH (see Park et al., 2000). For the conditional case, we have similar results where n is replaced by nh^d where d is the dimension of all the conditioning variables (Z, F_t), so $d = r + p + 2$ (see Jeong et al., 2010). So the rates of convergence of the conditional estimators are deteriorated by the dimension d.

In most of the empirical examples, a naive application of these nonparametric techniques may be problematic because real samples contain in general some anomalous data. In that case, the estimated frontier is fully determined by these outliers or extreme data points and the measurement of inefficiencies are totally unrealistic. Whereas most of the practitioners use a rule of thumb for outliers elimination, better approaches have been proposed in the frontier literature (Cazals et al., 2002; Daouia and Simar, 2007) to keep all the observations in the sample but to replace the frontier of the empirical distribution by (conditional) quantiles or by the expectation of the minimum (or maximum) of a subsample of the data. This

[4]We only focus the presentation on the output orientation version of the estimators, the same could be done for any other orientation (input, hyperbolic, directional distance).

latter method defines the order-m frontier that we will use here. To be short, the partial output-frontier of order-m is defined for any integer m and for an input x, as the expected value of the maximum of the output of m units drawn at random from the populations of firms using less inputs than x. Formally

$$\tau_m(x) = \mathbb{E}\left[\max(Y_1, \ldots, Y_m)\right], \tag{5}$$

where the Y_j are independently distributed as $S_{Y|X}(\cdot|X \leq x)$. The same applies for the conditional order-m frontier $\tau_m(x, z, f_t)$ where the Y_j are distributed as $S_{Y|X,Z,F_t}(\cdot|X \leq x, Z = z, F_t = f_t)$. Nonparametric estimators are obtained by plugging the nonparametric estimators of the survival functions in (5).

If m increases and converges to ∞, it has been shown (see Cazals et al., 2002) that the order-m frontier and its estimator converge to the full frontier, but for a finite m, the frontier will not envelop all the data points and so is much more robust than the FDH to outliers and extreme data points (see e.g. Daouia and Gijbels, 2011, for the analysis of these estimators from a theory of robustness perspective). Another advantage of these estimators is that they achieve the parametric rate of convergence $\sqrt{n}$ and that they have a normal limiting distribution.

2.2 The Location-Scale models

In this paper, for estimating the conditional measures, we will rather follow the approach suggested in Florens et al. (2014) which avoids direct estimation of the conditional survival function $S_{Y|X,Z,F_t}(y|x, z, f_t)$. As pointed by Florens et al., the procedure is less impacted by the curse of dimensionality (of the conditioning variables Z, F_t) and requires smoothing in these variables in the center of the data cloud and so avoiding smoothing at the frontier where typically the data are rather sparse and estimators are more sensitive to outliers. Moreover the inclusion of time factor $F_t = (t, X_{\cdot t}, Y_{\cdot t})$ enables us to eliminate the common time factor effect, in a very flexible nonparametric location-scale model. The statistical properties of the resulting frontier estimators are established in Florens et al. (2014).

We thus assume that the data are generated by the following nonparametric location-scale regression model

$$\begin{cases} X_{it} &= \mu_x(Z_{it}, F_t) + \sigma_x(Z_{it}, F_t)\varepsilon_{x,it} \\ Y_{it} &= \mu_y(Z_{it}, F_t) + \sigma_y(Z_{it}, F_t)\varepsilon_{y,it} \end{cases}, \tag{6}$$

where μ_x, σ_x and ε_x have each p components and, for ease of notations, the product of vectors is componentwise. So the first equation in (6) represents p relations, one for each component of X. We assume that each element of ε_x and ε_y have mean zero and standard deviation equal to 1. The model also assume that $(\varepsilon_x, \varepsilon_y)$ is independent of (Z, F_t).

This model allows us to capture for any (z, f_t), for each input, $j = 1, \ldots, p$ and for the output, the locations $\mu_x^{(j)}(z, f_t) = \mathbb{E}\left[(X^{(j)}|Z = z, F_t = f_t)\right]$, $\mu_y(z, f_t) = \mathbb{E}\left[(Y|Z = z, F_t = f_t)\right]$ and the scale effects $\sigma_x^{(j),2}(z, f_t) = \mathbb{V}\left[(X^{(j)}|Z = z, F_t = f_t)\right]$, $\sigma_y^2(z, t) = \mathbb{V}\left[(Y|Z = z, F_t = f_t)\right]$ of the environmental and common factors on the production plans.[5]

As explained in Florens et al. (2014), ε_x and ε_y can be interpreted as "pure" inputs and output, because due to the independence between the vector $(\varepsilon_x, \varepsilon_y)$ and (Z, F_t), they can be viewed as "whitened" versions of X and Y respectively. Since no particular assumption is made on the distribution of $(\varepsilon_x, \varepsilon_y)$, the model remains basically nonparametric. Note also that in the case where (Z, F_t) would be independent of all the inputs X and of the output Y, the functions μ_ℓ and σ_ℓ would be constant for $\ell = x, y$ and $(\varepsilon_x, \varepsilon_y)$ would simply be a standardized version of the original inputs and output.

The pure efficiency measure - that we derive below - provides a better indicator to assess the economic performance of production units over time and allows the ranking of production units affected by common shocks (captured by common factors f_t) and facing different environmental factors at different time periods (z_{it}).

To estimate the production frontier we follow the method in two stages proposed by Florens et al. (2014). In the first stage we estimate model (6) by using some usual nonparametric techniques (e.g. local constant or local linear): (i) estimation of the location functions $\mu_\ell(z_{it}, f_t)$ and (ii) estimation of the variance functions $\sigma_\ell^2(z_{it}, f_t)$ by regressing the square residuals, resulting from the location regression, on (z, f_t). For the location we use local linear and for the variance local constant to avoid negative values of the estimated variances. From this first analysis we obtain the residuals

$$\widehat{\varepsilon}_{x,it} = \frac{X_{it} - \widehat{\mu}_x(Z_{it}, F_t)}{\widehat{\sigma}_x(Z_{it}, F_t)}, \tag{7}$$

$$\widehat{\varepsilon}_{y,it} = \frac{Y_{it} - \widehat{\mu}_y(Z_{it}, F_t)}{\widehat{\sigma}_y(Z_{it}, F_t)}, \tag{8}$$

where for ease of notation, a ratio of two vectors has to be understood component wise. These are the whitened inputs and output obtained by eliminating the influence of the external and other environmental variables as common factors. In practice we will need to test the

[5]Hereafter, for a vector a, $a^{(j)}$ denotes its j^{th} component.

independence between $(\widehat{\varepsilon}_{x,it}, \widehat{\varepsilon}_{y,it})$ and (Z_{it}, F_t), i.e. the independence of whitened inputs and output from the external and global effects to validate the location-scale model (see Florens et al. 2014, for a bootstrap based testing procedure).

2.3 Estimation of the "pure" efficiencies

In the second stage, we can now estimate the production frontier for these whitened output and inputs and so we obtain for each observation (i, t) a measure of "pure" efficiency. This approach enables us to accommodate both time and cross-sectional dependence and obtain more reliable measure of efficiency. To some extent, the first step allows us also to control for endogeneity due to reverse causation between production process (labour, capital and output) and external variables (in our case FDI). Moreover, as pointed by Florens et al. (2014), by cleaning external factors dependence in the first stage, we avoid the problem of curse of dimensionality due to the dimension of the external variables when estimating the production frontier.

In practice this leads to estimate the attainable set of pure inputs and output $(\varepsilon_x, \varepsilon_y)$. The latter is defined as

$$\Psi_\varepsilon = \left\{(e_x, e_y) \in \mathbb{R}^{p+1} | H_{\varepsilon_x,\varepsilon_y}(e_x, e_y) = \mathrm{Prob}(\varepsilon_x \leq e_x, \varepsilon_y \geq e_y) > 0\right\}.$$

The nonparametric FDH estimator is obtained by plugging the empirical estimators $\widehat{H}_{\varepsilon_x,\varepsilon_y}(e_x, e_y)$ obtained with the observed residuals defined in (7) and (8). As shown in Florens et al. (2014), replacing the unobserved $(\varepsilon_x, \varepsilon_y)$ by their empirical counterparts $(\widehat{\varepsilon}_x, \widehat{\varepsilon}_y)$ does not change the usual statistical properties of frontier estimators. So we have the consistency for the full-frontier FDH estimator and $\sqrt{n}$-consistency and asymptotic normality for the robust order-m frontiers. It is conjectured in Florens et al. (2014), that if the functions μ_ℓ and σ_ℓ for $\ell = x, y$, are smooth enough, the conditional FDH estimator would keep its usual nonparametric rate of convergence i.e. $n^{1/(p+1)}$.

A "pure" measure of efficiency can then be obtained by measuring the distance of a particular point $(\varepsilon_{x,it}, \varepsilon_{y,it})$ to the efficient frontier. Since the pure inputs and output are centered on zero, they may have negative values and so radial distances are inappropriate. We should rather use directional distances defined for a particular unit (e_x, e_y) as

$$\delta(e_x, e_y; d_x, d_y) = \sup\{\gamma | H_{\varepsilon_x,\varepsilon_y}(e_x - \gamma d_x, e_y + \gamma d_y) > 0\}, \tag{9}$$

where $d_x \in \mathbb{R}^p_+$ and $d_y \in \mathbb{R}_+$ are the chosen direction. In our case here we choose an output orientation so that $d_x = 0$ and we can choose $d_y = 1$, for more general cases, see Simar and Vanhems (2012) (if only some elements of $d_x = 0$ see Daraio and Simar, 2014 for practical computations). So, for this particular output direction and in the case of univariate output we follow here, the optimal production frontier can be described at any value of the pure input $e_x \in \mathbb{R}^p$, by the function

$$\varphi(e_x) = \sup\{e_y | H_{\varepsilon_x,\varepsilon_y}(e_x, e_y) > 0\}, \tag{10}$$

so that the distance to the frontier of a point (e_x, e_y), in the output direction, is directly given by $\delta(e_x, e_y; 0, 1) = \varphi(e_x) - e_y$. Then, for each units in the sample, the "pure" efficiency estimator is obtained through

$$\widehat{\delta}(\widehat{\varepsilon}_{x,it}, \widehat{\varepsilon}_{y,it}; 0, 1) = \widehat{\varphi}(\widehat{\varepsilon}_{x,it}) - \widehat{\varepsilon}_{y,it}, \tag{11}$$

where $\widehat{\varphi}(\cdot)$ is the FDH estimator of the pure efficient frontier in the output direction. It is simply obtained as

$$\begin{aligned} \widehat{\varphi}(e_x) &= \sup\{e_y | \widehat{H}_{\varepsilon_x,\varepsilon_y}(e_x, e_y) > 0\} \\ &= \max_{\{(i,t) | \widehat{\varepsilon}_{x,it} \leq e_x\}} \widehat{\varepsilon}_{y,it}. \end{aligned} \tag{12}$$

Similar expressions can be derived for the order-m efficiency estimator. As explained above, the order-m frontier at an input value e_x, is the expected value of the maximum of the outputs of m units drawn at random in the population of units such that $\varepsilon_{x,it} \leq e_x$. The nonparametric estimator is obtained by looking to its empirical version:

$$\widehat{\varphi}_m(e_x) = \widehat{E}\left[\max\left(\varepsilon_{y,1t}, \ldots, \varepsilon_{y,mt}\right)\right], \tag{13}$$

where the $\varepsilon_{y,it}$ are drawn from the empirical conditional survival function $\widehat{S}_{\varepsilon_y|\varepsilon_x}(e_y | \widehat{\varepsilon}_{x,it} \leq e_x)$. This can be computed by Monte-Carlo approximation or by solving a univariate numerical integral (for practical details see Simar and Vanhems 2012).

It is also possible to recover the conditional output-oriented frontier in the original units of the inputs and output. It is directly obtained at any value of (x, z, f_t) as

$$\tau(x, z, f_t) = \mu_y(z, f_t) + \varphi(e_x)\sigma_y(z, f_t), \tag{14}$$

where e_x is the p-vector with components $(x - \mu_x(z, f_t))/\sigma_x(z, f_t)$. In terms of estimates, this gives for a particular point (x_{it}, z_{it}, f_t) the estimated frontier point in the original units

$$\widehat{\tau}(x_{it}, z_{it}, f_t) = \widehat{\mu}_y(z_{it}, f_t) + \widehat{\varphi}(\widehat{\varepsilon}_{x,it})\widehat{\sigma}_y(z_{it}, f_t). \tag{15}$$

By using (6) and (11) above we see that this can be equivalently written as

$$\widehat{\tau}(x_{it}, z_{it}, f_t) = y_{it} + \widehat{\delta}(\widehat{\varepsilon}_{x,it}, \widehat{\varepsilon}_{y,it})\widehat{\sigma}_y(z_{it}, f_t), \tag{16}$$

which has a nice interpretation: we see that the directional distance from the observed input-output point (x_{it}, y_{it}) facing external conditions (z_{it}, f_t) to the efficient frontier is given by the "pure" efficiency measure evaluated at the pure input-outputs $(\widehat{\varepsilon}_{x,it}, \widehat{\varepsilon}_{y,it})$ rescaled by the local standard deviation $\widehat{\sigma}_y(z_{it}, f_t)$. Finally if Farrell-type efficiency estimates are wanted, as in (4), an estimate is given by

$$\widehat{\lambda}\,(x_{it}, y_{it}|z_{it}, f_t) = \frac{\widehat{\tau}(x_{it}, z_{it}, f_t)}{y_{it}} \geq 1, \tag{17}$$

with equality to 1 for points on the estimated conditional frontier (having pure efficiency $\widehat{\delta}$ equal to zero).

Note that when back to original coordinates, we are back to the curse of dimensionality, typically the n appearing in the convergence rates for the frontier estimates in the "pure" units is replaced by nh^d where d is the dimension of all the conditioning variables (Z, F_t) (see Florens et al. 2014, for details).

2.4 Effect of Z on the production process

It should be noticed that for comparing the performance of units, and proceed to rankings, the efficiency measures in original units are not appropriate, because they measure the efficiency scores of units relative to different frontiers, according the current values of (z, f_t). In the setup we develop here, the only way to compare the efficiency scores is when analyzing the performance of the units in the space of "pure" inputs and outputs, as described above. However the conditional measures will be useful to investigate the impact of the conditioning variables on the production process, by comparing the conditional measures $\lambda(x, y|z, f_t)$ with the unconditional measures $\lambda(x, y)$. We follow the procedure described in details in Bădin et al. (2012). In what follow, since we want to analyze the potential effect of FDI on the production process, we will compare estimates of $\lambda(x, y|z, f_t)$ with those of $\lambda(x, y|f_t)$. The procedure allows to disentangle the potential effects of FDI on the boundary (shift of the frontier) and on the distribution of the inefficiencies.

The first effect can be investigated by considering the ratios of conditional to unconditional efficiency measures, which are measures relative to the full frontier of respectively, the conditional and the unconditional attainable sets. Note that since in our case y is univariate, these ratios are the same as the ratios of the frontier levels. So we have

$$R_O(x, y|z, f_t) = \frac{\lambda(x, y|z, f_t)}{\lambda(x, y|f_t)}. \tag{18}$$

By construction, for the output orientation, $R_O(x, y|z, f_t) \leq 1$ (the conditional efficient boundary is below the unconditional one) and $R_O(x, y|z, f_t) = 1$ if and only if, at time t, there is no shift of the efficient boundary of the two attainable sets due to z. Looking to these ratios as a function of z =FDI allows to investigate the effect of FDI on this potential shift.[6] A global tendency of the ratios to increase with the conditioning variables indicates a favorable effect (the conditional efficient frontier moves up to the unconditional one when the variables increase, i.e. the variables act as freely available inputs) and unfavorable in the opposite case (the conditional efficient boundary moves away form the marginal one when the variables increase, the variables act as undesirable outputs). As illustrated in Daraio and Simar (2007), some extreme or outlying data points may hide the real effect of Z, so it is suggested to do the same analysis with our order-m frontier, with large values of m to get robust estimates of the full frontier (we discuss in the application how to select m for this purpose). In this case, the ratios to be analyzed are given by

$$R_{O,m}(x, y|z, f_t) = \frac{\lambda_m(x, y|z, f_t)}{\lambda_m(x, y|f_t)}. \tag{19}$$

As pointed in Bădin et al. (2012), the full frontier ratios, or their robust version with large values of m, indicate only the influence of Z on the shape of the frontier, whereas the partial frontiers for small values of m, characterizes behavior of the shift more in the center of the distribution of efficiencies, inside the attainable sets. For instance if $m = 1$, the order-m frontier turns out to be an average production function and the ratios (19) would analyze the shift of the mean of the distribution of the inefficiencies. Some potential shifting effect already observed with (18) could be enhanced (or reduced) if the effect is different with the

[6]Because the effect could be different for different values of X (possibility of interactions), the analysis of these ratios has to be done for fixed levels of the inputs x, as suggested in our application.

ratios (19). As explained in Bădin et al. (2012), the ratios are not bounded by 1, because the order-m efficiency scores are not bounded by 1. The latter equal to 1 if and only if (x, y) is on the m-frontier, bigger than 1, if they are below the m-frontier and smaller than 1 if they are above the m-frontier. But, as for the full ratios above, a tendency of $R_{O,m}(x, y|z, f_t)$ to increase with the conditioning variables indicates a favorable effect of these variables on the distribution of the efficiencies (the conditional distribution is more concentrated to its upper boundary when the conditioning variables increase) and the opposite in the case of a unfavorable effect. If this effect is similar to the one shown with the ratios with full frontier, we can conclude that we have a shift of the frontier while keeping the same distribution of the efficiencies when the conditioning variable Z change; if the effect with the partial frontiers is more important than for the full frontier, this indicates that in addition to a shift of the frontier, we have also an effect on the distribution of the efficiencies.

2.5 Parametric fit of the nonparametric frontiers

It has been argued that parametric models provide much richer interpretations of the production process in terms of elasticities, etc. This is true if the chosen parametric model is a reasonable approximation of the true frontier. On the other hand, and as discussed in details in Florens and Simar (2005), most of the methods of estimation of these parametric frontier models suffer from some drawbacks, in case of heterogeneity of the efficiency distribution over the input values and /or in case of outlying data points.

If a researcher want to fit a particular parametric model to the frontier, Florens and Simar (2005) suggest an approach that address most of the drawbacks of the usual methods and provide robust fits of the frontier. Suppose we want to see if a parametric model (e.g. Cobb-Douglas) is appropriate, Florens and Simar propose to project all the input-output data points on the FDH frontier or even better on the robust order-m frontier and then adjust the chosen parametric model to this cloud of "efficient" points, e.g. by simple OLS (ordinary least squares). Florens and Simar (2005) analyze the statistical properties of the resulting estimators and show that they are consistent estimators of the pseudo-true values of the parameters. The pseudo-true values are the values of the parameters given by best approximation of the true unknown frontier by the selected parametric model (in the sense of integrated squared errors). If the selected parametric model (like Cobb-Douglas) is true, these pseudo-true values are the true values of the parameters. If we use the order-m robust version we have even $\sqrt{n}$-consistency and limiting unbiased normal distribution with a variance, that can be estimated by bootstrap techniques. The analysis of traditional goodness of fit measures would help the researcher to assess if the chosen parametric model is a reasonable approximation.

In our case here, we will adjust a parametric frontier in the "pure" inputs-output space (for all the reasons explained in the preceding section). The advantage of the Florens-Simar semiparametric approach with respect to a standard parametric one, is that in the first step it relies on a fully nonparametric model and only search for the best parametric approximation; second, the method does not require any parametric assumption regarding the distribution of the inefficiencies. In particular, the assumption of complete homogeneity of considered economic units is not needed. Therefore the economic units under investigation can potentially consist of different groups of populations governed by different distributional laws of the generation of input-output mix and on efficiency. This means that, if the sample is formed by developed and developing countries, as in our case, these groups of countries can have different distributions of efficiency scores. This enables us to analyze the world production technology and have a direct economic interpretation in terms of elasticity and technology progress. In addition when using the order-m approach we are robust to outliers and extreme data points.

We can for example estimate the following Cobb-Douglas parametric frontier model (in a linear form because we assume units of measurement are already in log)

$$\widehat{\varepsilon}^{\delta}_{y,it} = \alpha + \beta' \widehat{\varepsilon}_{x,it} + \zeta_{it} \tag{20}$$

where $\widehat{\varepsilon}_{x,it}$ and $\widehat{\varepsilon}^{\delta}_{y,it} = \widehat{\varphi}(\widehat{\varepsilon}_{x,it})$ (or $\widehat{\varphi}_m(\widehat{\varepsilon}_{x,it})$ if robust versions are wanted) were defined above and ζ_{it} is the fitting noise. We obtain the estimated fitted Cobb-Douglas production frontier $\widehat{\widehat{\varepsilon}}^{\delta}_{y,it} = \widehat{\alpha} + \widehat{\beta}' \widehat{\varepsilon}_{x,it}$. If we come back to the original units, the estimated Cobb-Douglas frontier parametric model is thus given by (we use the shortcut notation $w_{it} = (f_t, z_{i,t})$):

$$\widehat{y}^{\delta}_{it}(x_{it} \mid w_{it}) = \mu_y(w_{it}) + \sigma_y(w_{it}) \widehat{\widehat{\varepsilon}}^{\delta}_{y,it} \tag{21}$$

It can be shown that this can be written as

$$\widehat{y}^{\delta}_{it}(x_{it} \mid w_{it}) = \widetilde{\alpha}(w_{it}) + \widetilde{\beta}'(w_{it}) x_{it}, \tag{22}$$

where (using again for ease of notations, the component wise division of vectors)

$$\widetilde{\alpha}(w_{it}) = \mu_y(w_{it}) + \sigma_y(w_{it}) \left[\widehat{\alpha} - \widehat{\beta}' \frac{\mu_x(w_{it})}{\sigma_x(w_{it})} \right] \tag{23}$$

$$\widetilde{\beta}(w_{it}) = \sigma_y(w_{it}) \frac{\widehat{\beta}}{\sigma_x(w_{it})} \tag{24}$$

To assess the characteristics in terms of technology, capital and labour elasticities of the production frontier, we can look at the exponential of the latter coefficients as a function of time (see below in the application). Technological changes is captured by $\widetilde{\alpha}(w_{it})$ which indicates if the world frontier itself has moved outward (progress) or inward (regress), or both over time. The evaluation over time of the components of $\widetilde{\beta}(w_{it})$ enables us to assess if technology over time has been more capital or labour deepening and, hence, to appraise policy implications in favor of capital or labour accumulation in less developed countries to promote convergence towards richest ones.

3 Empirical Application

3.1 The data and the variables

Our non parametric approach in constructing the worldwide production frontier does not require the specification of the production functional form, and also limit the problem of 'curse of dimensionality' at the second stage of our methodology. In addition, we provide an analysis which is robust to extreme or outlying data points that might hide some features of the production process. We consider the simplest production model with only three macroeconomic variables: aggregate output and two aggregate inputs (labour and capital).

The dataset is collected over the period, 1970-2007 (38 years) for a total of 44 countries using data from the Penn; 26 are developed OECD countries (Australia, Austria, Belgium, Canada, Chile, Hong Kong, Denmark, Finland, France, Germany, Greece, Ireland, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Turkey, the United Kingdom and the United States) and 18 are developing countries (Argentina, Bolivia, Côte d'Ivoire, Dominican Republic, Ecuador, Honduras, Jamaica, Kenya, Madagascar, Malawi, Morocco, Nigeria, Panama, Philippines, Thailand, Venezuela, Zambia, Zimbabwe).[7]

[7] The choice of countries depends on data availability. Developed and developing countries are classified following the World Bank (2007) classification. See Mastromarco and Simar (2014) for a detailed data

Using data from the Penn World Tables (version 6.3) we calculate as measure of output the real gross domestic product and it is obtained as $RGDPCH * POP$, where $RGDPCH$ is per capita real GDP computed via the chain method, POP is the population. The resulting output is GDP measured in million US dollars at 2005 constant prices. For labor input, we use the number of workers as $RGDPCH * POP/RGDPWOK$, where $RGDPWOK$ is real GDP per worker. For the capital input, we proceed as follow. Real aggregate investment in million US dollars at 2005 constant prices is computed as $I = RGDPL * POP * KI$, where $RDGPL$ is the real GDP, and KI is the investment share of real GDP. Capital K which is our chosen input is then measured in million US dollars at 2005 constant prices and constructed applying the perpetual inventory method (PIM) by using the real investment series.[8] All three variables are rescaled to get a standard deviation of 1 and then transformed in logarithms before estimation.

For globalization factor we identify one of the most important channels: FDI inflows, measured as net inflows of foreign direct investment, which are then transformed as a ratio to GDP.[9]

Our external variable FDI might suffer from endogeneity bias. The endogeneity caused by reverse causality is still an open issue in the empirical studies investigating the relationship between total factor productivity (TFP) and FDI. In this paper we explicitly address this issue by eliminating in the first stage the dependence of the FDI on the production process. Furthermore, the global economy becomes increasingly integrated, all the individual countries are likely to be exposed more to global shocks. As explained above we follow Pesaran (2006) and Bai (2009) and consider $F_t = (t, X_{\cdot t}, Y_{\cdot,t})$ as proxy for these common factors.

3.2 Pure efficiency analysis

Now we can follow the two-stage estimation procedure described above which enables us, in the first stage, to better capture the impacts of global shocks (such as FDI, trade policy and

description.

[8]PIM is necessitated by the lack of capital stock data across all the countries. For an individual country, the capital stock is constructed as $K_t = K_{t-1}(1-\theta)+I_t$, where I_t is investment and θ the rate of depreciation assumed to be 6% (*e.g.*, Hall and Jones, 1999; Iyer *et al.*, 2008). Repair and maintenance are assumed to keep the physical production capabilities of an asset constant during its lifetime. Initial capital stocks are constructed, assuming that capital and output grow at the same rate. Specifically, for country with investment data beginning in 1970, we set the initial stock, $K_{1970} = I_{1970}/(g+\theta)$, where g is the 10-year output growth rate from 1970 to 1980. Estimated capital stock includes both residential and non-residential capital.

[9]FDI is sourced from the World Bank World Development Indicators and Unctad, all the other data from PWT 6.3. The observation period is selected by the data availability.

cycle fluctuations) and, hence, CSD on the world production frontier and technical efficiency. By applying the estimation of the models (6) to our transformed data, we obtain by equations (7) and (8) the “pure” versions of our inputs , $\widehat{\epsilon}_{X1}$ (Capital) and $\widehat{\epsilon}_{X2}$ (Labour) and of the output $\widehat{\epsilon}_{Y}$ (GDP). For the 1672 observations of our sample these values are displayed in Figure 1. As expected, we see indeed in these “pure” units, that we have an increasing relationship between the production output and labour and capital inputs.

Before looking for frontier estimates we have to test if the “pure” inputs-output $(\varepsilon_x, \varepsilon_y)$ are independent of the conditioning variables (F_t, Z). For checking this we apply the bootstrap test described in Florens et al. (2014). The test statistics is of the Kolmgorov-Smirnov type and compare the joint cdf of $(\widehat{\varepsilon}_x, \widehat{\varepsilon}_y, F_t, Z)$ with the product of the marginal cdf of $(\widehat{\varepsilon}_x, \widehat{\varepsilon}_y)$ and of (F_t, Z). Bootstrap methods can be used to obtain critical values. We know from the literature Einmahl and Van Keilegom (2008) and Neumeyer (2009) that the bandwidths for determining $(\widehat{\varepsilon}_x, \widehat{\varepsilon}_y)$ should be a smaller order than the optimal bandwidths determined by least-squares cross validation. We follow here the procedure suggested in Florens et al. and we computed the p-value of the null hypothesis (independence) scaling the optimal bandwidths by a factor c ranging from 0.25 to 1. Figure 2 show the results based on 2000 bootstrap replications. We see that the p-values range from 0.2 to 0.5 and so this does not provide any evidence to reject the null hypothesis of independence. We conclude that our first stage location-scale models was able to clean the effect of (F_t, Z) on the original inputs and output, confirming that the influence of FDI and the cross section dependence has been removed from our data.

The estimation of the world production frontier then follows in the second stage. The full frontier estimate is the FDH of the preceding cloud of points shown in Figure 1. It was defined above as $\widehat{\varphi}(e_x)$ and is displayed in the left top panel of Figure 4 below.

For a robust version of the frontier we have to specify a value for m. We choose the procedure advocated in several papers (see e.g. Simar, 2003 or Daraio and Simar, 2007). We compute the order-m frontier for several values of m and look to the corresponding percentage of data points staying above the resulting $\widehat{\varphi}_m(e_x)$. We know that this percentage decreases when m increases, converging to 0 when $m \to \infty$ since at a moment, for very large value of m we will observe $\widehat{\varphi}_m \equiv \widehat{\varphi}$, i.e. the FDH estimate. This percentage of points outside the frontier (with values $\widehat{\varepsilon}_{y,it} > \widehat{\varphi}_m(\widehat{\varepsilon}_{x,it})$) as a function of m is shown in Figure 3. We see that as expected for small values of m this percentage decreases rapidly but around values near 1000, the value of m has to increase a lot to get the remaining points outside the frontier at this stage, under the frontier. So the points that are outside the frontier for $m = 1000$ are rather extreme and may be outlying relative to the rest of the clouds. In fact the “elbow” effect just described is more precisely identified near $m = 800$.[10] So for the robust version of the full frontier, we select $m = 800$ (this leaves 8.5% of data points above the corresponding order-m frontier). The resulting order-m frontier is displayed at the right top panel of Figure 4: we see that the two frontiers, full and order-m, are globally very similar except for some local values of the inputs. So in our data set here, the outlying points are not too influential globally, but they may influence the efficiency score of some units.

The distribution of estimated inefficiency in Figure 5 reveals most of the OECD countries under analysis as globally efficient over the period, since the histogram has an exponential shape with most of its mass near the efficiency level 0 and some rare very inefficiency observations. We note that the order-m inefficiencies are quite similar confirming the above analysis; we see here the 8.5% of data (142 units) above the order-m frontier with negative $\widehat{\delta}_m$'s, but not so far from the frontier except a few observations. This explains why these are not so influential globally and why both results, full and order-m, are so similar.

Table 1 summarizes descriptive statistics for “pure” order-m efficiency for the 44 countries. We find that the most efficient countries over the sample period are the United States (USA), Japan (JPN), Germany (GER) and Ivory Coast (CVI), while the least efficient are Chile (CHL), Zambia (ZMB), Philippines (PHL) and Nicaragua (NGA). Positive performance observed in Ivory Coast is also documented in other studies (see Koop et al. 2000).

Ivory Coast has, for West Africa region, a relatively high income per capita (USD 1014.4 in 2013) and plays a key role in transit trade for neighboring, landlocked countries. The country is the world's largest exporter of Cocoa beans, and the fourth largest exporter of goods, in general, in sub-Saharan Africa (following South Africa, Nigeria and Angola). As the second largest economy in West Africa and a top world exporter of cocoa and cashews, Ivory Coast boasts enormous economic potential. Macroeconomic performance continued to be impressive in 2013, with economic activity expanding by an estimated 8.7%. Inflation remained subdued at 2.5%. The macroeconomic prospects for 2014 remain positive, especially given the expectations of a vigorous growth rate and low inflation. Continued strong macroeconomic performance and further progress on the government's structural reform program is necessary in order to support GDP growth, improve living standards for the most vulnerable populations, and allow Ivory Coast to transform itself into an emerging economy.[11]

To give a visual impression of the change in “pure” efficiency over time, “pure” efficiency

[10] The order-m frontiers with $m = 800$ and $m = 1000$ are very similar.

[11] Country Report 2014, The World Bank Group, web site http://www.worldbank.org/afr/.

for each year is displayed in Figure 6 for the USA, Japan, Germany (the best countries in our analysis) Nicaragua, Philippines, Zambia (the worst performing countries), and for Belgium and Italy. Note also that we use the Hodrick and Prescott (1996) filter to smooth the time paths with a smoothing weight equal to 100.

By applying spectral analysis (Mastromarco and Woitek 2007) we examine the business cycles of our pure measure of efficiency which gives insights on prevailed cycles of the technological catch-up process of the countries under analysis. Table 2 describes the relative importance of efficiency cycles. The columns report the estimated variance shares in the frequency bands, i.e. the cycles with a length of 3-5 years (the Kitchin cycle), 5-7 years and 7-10 years (the Jugular cycle).[12] The dominant frequencies contain important information of the structure of efficiency. The efficiency is dominated by the shorter cycle of 3-5 years and 5-7 years cycle for all countries, except Austria, Ivory Coast, Dominican Republic, Greece, Jamaica, Morocco, New Zeeland, Panama, Spain, Zambia.

3.3 Influence of FDI on the production process

To assess the influence of FDI on the production process, we investigates the ratios of conditional and unconditional efficiency measures for full and partial frontier as discussed in Section 2.4. Figure 7 displays all the needed results. We computed the ratios (18) for the full ratios and the ratios (19) for $m = 800$ (robust version of the full ratios) and for $m = 1$ to assess the influence of FDI on the average of the inefficient distribution. In addition, since there may be some interaction with the values of the inputs, we fix these values of Labor and Capital simultaneously at their three univariate quartiles, looking to the effect of FDI on the production process for small, medium and large countries, in term of their inputs.[13] To facilitate the interpretation of the pictures we computed as usual in this kind of analysis the nonparametric regression line of the ratios on FDI.

The main messages of these pictures is as follows. To investigate the effect of FDI on the shift of the frontier, we have to analyze the ratios for the full frontier, and its robust version. First we see that the order-m results are very similar to the full frontier results, this confirms again, than in our data set, the most outlying points are not too influential for global analysis. Second, we see for left and right panels an inverted-U shaped of the regression lines for small and large countries, and a more linear shape for the medium sized

[12] The traditional business cycle ranges are 3-5 years (Kitchin cycle), 5-7 years and 7-10 years (Juglar cycle).

[13] We could provide more pictures with more values and combinations of the inputs but to save space, we limit our analysis to these three combinations.

countries. Third and importantly, the level of the ratios is changing with the size of the countries: low level, near 1 for the small, increasing a lot for medium and large countries (near the values 2 or 3). In our setup this can be interpreted as follows.

We might have some shift of the frontier when FDI increases, but with a decreasing effect at large values of FDI. So, FDI acts on the shift of the boundary. Hence, from this evidence, FDI appears to play an important role in accelerating the technological change (shifts in the frontier). This result seems to confirm the theoretical hypothesis that FDI leads to increase in productivity by spurring competition: foreign firms have to invest even more in innovation in order to keep up with their technological advantage (Glass and Saggi 1998). This is particularly true for medium and big countries (large values of ratios). The evidence of decreasing returns of FDI can be easily explained by the adjustment costs involved in FDI, e.g. Tybout (1992) and Coe and Helpman (1995).

The bottom panels of Figure 7 deserves also some comments, they allow, when compared to the top panels, to identify some changes in the distribution of the inefficiencies due to FDI. Globally, we cannot see big changes in the shape of the clouds of points and of the regression lines, even if the shape is more linear for large countries. But the level of the ratios are quite different for medium and large countries: now in all the cases, the ratios remains not far from 1. So the shift of the technology we have identified above, is compensated by the fact that for medium and large size countries, there is much more dispersion, ending up with similar values for the average production levels. This could be interpreted as he fact that FDI induces some shift of the production frontier, but not necessarily catching-up.

Efficiency is the most important growth component for convergence analysis of countries that are below the technological frontier because it reflects "the process of imitation and transmission of existing knowledge" (Romer 1986). Quah (1997), Mankiw et al. (1992), Barro and Sala-i-Martin (1995) argue that slow convergence in the level of output per worker is caused by slow technological catch-up. FDI might increase efficiency and, hence, convergence. This occurs with the adoption of foreign technology through technology licensing or technology purchase, imports of high technology capital goods, and the skills acquired by the local labour force as they are trained by the foreign firms (Borensztein et al. 1998, De Mello 1999, Xu 2000). However, our findings support the divergent evolution of output among countries with respect to FDI.

3.4 Cobb-Douglas approximation

As explained in Section 2.5, if some parametric approximation of the production frontier is reasonable, this may facilitate the analysis and the interpretations in terms of elasticities, etc.... It could provide a complementary analysis to the nonparametric one suggested so far. The idea is to investigate again, in a parametric setup, the channels through which FDI affects the production process whether through technological change - shifts of the frontier - or through factor accumulation. Therefore we estimate a Cobb-Douglas production frontier by projecting all data points, in the "pure" units on the FDH frontier (or its robust version, the order-m, with $m = 800$) and then using simple OLS to fit the Cobb-Douglas, since the original data have already been transformed in logarithms.

Table 3 reports the results for the estimates of the constant (α) and of the output elasticities of capital (β_1) and labour (β_2) of the world frontier. We display also the results obtained by the more classical parametric estimator called COLS, or shifted-OLS, in the parametric frontier literature (see e.g. Kumbhakar and Lovell, 2002). We see again, as in the preceding analysis, that the full and the order-m frontiers give quite similar Cobb-Douglas approximations. Since the latter have better known asymptotic properties, we will rely the interpretation below in terms of the results obtained with the order-m frontier. The last column of the table gives $R^2 = 0.89$ indicating that the Cobb-Douglas fit is quite reasonable (note that for the COLS, the high value of the R^2 cannot be directly compared because it is the R^2 derived from the first step OLS estimation of the average production function, in the center of the cloud of points $(\widehat{\varepsilon}_{x,it}, \widehat{\varepsilon}_{y,it})$ before the shift of the average regression line to the frontier). We see also that the result of the level of the frontier, α obtained by COLS is quite different from these obtained by the Florens and Simar approach. We will see below that the COLS estimate is irrelevant in our application. Note that here we are able to produce the resulting Cobb-Douglas fit in the "pure" units, this provides the bottom panels of Figure 4. Looking carefully we see that indeed the approximation of the full frontier is slightly above the order-m one, but that they give roughly the same results, as already commented.

We see also that the technology of the world frontier reveals that output is elastic especially with respect to labour (about 0.70), while the output elasticity with respect to capital is lower (around 0.30).[14] The sum of the output elasticities is approximately equal to one indicating that a constant returns to scale (CRS) assumption seems reasonable. This enables us to interpret the world production technology in the $y = Y/L$ and $k = K/L$ space,

[14]Labour contributions are higher as expected, implying that it is easier to maintain output and profitability by reducing employment or increasing labour productivity rather than by dismissing capital stocks.

allowing picture in two dimensions. Figure 8 display the results of the three Cobb-Douglas fit in this space, where we deloged the units (so we took the exponential of the log-linear model estimated above). The Figure 8 is for the full data set and we see clearly that in our application, the COLS method collapses. The latter is indeed based on an assumption of homoskedasticity that is not reasonable here and the COLS fit envelops, by construction, all the data points and so is determined by a single outlying point (near coordinates (0.5,3) in the figure). However the estimates obtained by the Florens and Simar approach give very reasonable frontier, enveloping most of the data points, but being much more robust to extreme or outlying data points.

To investigate changes over time, we can first have a look to Figure 9, or any other similar picture. Here we compare the position of the data points at two different periods with respect to the world frontier (which is the same for all the years). Here we choose to compare the first (1970) and the last years (2007). A careful analysis (increasing the size of the figure) allow to see the evolution of particular countries and so appreciate their change in their pure inefficiency.

In 1970 at very low capital-labour ratios, it appears to be technological frontier countries as US and Canada and, surprisingly, also very poor country as Jamaica. The last year 2007, displays an outward shift in frontier for higher but still quite low capital-labour ratios as Germany and the UK, very little change in the frontier for the middle of the distribution of capital-labour ratios and a sizeable expansion of potential output at very high capital-labour ratios as the Netherlands (not the large technological-change for the New Zeeland).

In addition, the comparison of the pictures at the two periods indicates that there is slightly less dispersion of the data and that in both years, the frontier is typically determined by countries with low K/L ratios.

The frontier countries - the US, Germany and Canada - indicate that production technology on the frontier is capital saving (labour using). The evidence confirms that the world frontier is at low level of capitalization. These pictures displays that technology change for the frontier countries over this period has been nonneutral. In particular, Hicks-neutral technological change would shift the frontier in the $y = Y/L$ and $k = K/L$ space vertically by the same proportional amount at all capital-labour ratios. The change in the technologies between 1970 and 2007 for the frontier countries seems to be consistent with Harrod-neutral (labour-augmenting) technological change.[15]

[15] Harrod-neutral technological change would shift the frontier countries radially (i.e., by equal proportional factors along rays from the origin).

Another advantage of the use of a parametric approximation (if it appears to be reasonable, as in our case) is that it allows easiest interpretation due to the presence of parameters having their own economic interpretation. Here we will reintroduce time and FDI dependence by looking to the parameters of the Cobb-Douglas in original units as given by (23) and (24). The time variation in the technology and output elasticity of capital and labour is displayed in Figure 10. We do not see particular structure here because the effect of time is mixed with the effect of FDI. More interesting is to look for the time-variant technology and factors output elasticity for fixed values FDI as illustrated in Figure 11. So we analyzed the evolution over time of the Cobb-Douglas parameters over time for FDI fixed at its three quartiles for each time period. The bottom panel of this figure reveals a positive effect of FDI on constant, this indicates indicates a favourable effect of FDI on the production process via technological change. This result seems to confirm the theoretical hypothesis that FDI leads to increase in productivity by spurring competition: foreign firms have to invest even more in innovation in order to keep up with their technological advantage (Glass and Saggi 1998). This also complements the findings from our nonparametric approach above.

On the contrary the top panel of Figure 11 demonstrates that FDI does not impact the output elasticity of capital whereas there is a visible scale effect on labour elasticity as shown in the middle panel, and this effect which is higher for higher level of FDI. This evidence suggests that high level of foreign direct investments are substitute with high level of domestic capital investments and complement with labour.

This confirms the conjecture that a nation needs to have well trained and skilled labour force with a high productivity for knowledge diffusion through FDI, which 'supports the view of complementarities between disembodied knowledge of multinational firms and the absorptive capacity in host countries' (Wijeweera et al. 2010, Mastromarco and Ghosh 2009). For multinational corporations operating in skill-intensive industries, the level of human capital acquired by training is very important (Blomstrom and Kokko 1997, Miyamoto 2003).

Our findings reveal that FDI influences positively the production process through different channels as technological changes and scale effects. This proves that knowledge embodied in FDI is transferred for technology externalities (shifts of the frontier). Hence, from this evidence, FDI appears to play an important role in accelerating technological change (shifts in the frontier). This result corroborates the theory that FDI increases productivity by stimulating competition and inducing multinational firms to invest more in innovation (Glass and Saggi 1998). To a lesser extent, FDI can also increase factor accumulation by influencing output elasticity. This occurs with the adoption of foreign technology through technology licensing or technology purchase, imports of high technology capital goods, and the skills acquired by the local labour force as they are trained by the foreign firms (Borensztein et al. 1998, De Mello 1999, Xu 2000).

4 Conclusion

The productivity analysis recognizes the importance of considering the spillover effects of global shocks and business cycles due to increasing globalization and interconnection among countries. So far all studies analyzing effect of common external factors on productivity of countries have been on the stream of parametric modelling which suffers of misspecification problems when the data generating process is unknown, as usual in the applied studies.

We propose the unified non parametric framework for accommodating simultaneously the problem of model specification uncertainty, potential endogeneity and cross-section dependence in modelling technical efficiency in frontier models. In particular, we adopt the two-step procedure advanced by Florens et al. (2014), which enables us to deal with both endogeneity and cross section dependence jointly by combining location scale model and conditional efficiency estimation to eliminate the dependence of production inputs/outputs on the common factors. Our non parametric approach to estimate conditional efficiency does not require any parametric assumption regarding technology or efficiency term. Moreover, the assumption of complete homogeneity of considered units is not needed. Therefore the economic units under investigation, can potentially consist of different groups of population governed by different distributional laws of the generation of input-output mix and on efficiency. This is an advantage in our sample formed by developed and developing countries which most likely have different distributions of efficiency scores.

Moreover our frontier model enables us to see whether the effect of environmental/global variables on productivity occurs via technology change or efficiency. We can then quantify the impact of environmental/global factors on efficiency levels and make inferences about the contributions of these variables in affecting efficiency.

We find that the most efficient countries over the sample period are the United States, Japan and Germany, while the least efficient are Chile, Zambia, Philippines and Nicaragua. By applying spectra analysis we conclude that the business cycles of our pure efficiency is of length 3-5 years and 5-7 years for most of the countries under analysis.

According to the literature (Borensztein et al. 1998, De Mello 1999, Xu 2000, Mastromarco and Ghosh 2009, Iyer et al. 2008), one of the main channels through which the foreign technology diffusion occurs is through foreign direct investment. Our paper extends previous studies on similar topics by investigating this channel in full nonparametric framework which avoids some restrictive and often unverifiable prior assumptions on functional relationships and distributions.

We focus on the effect of FDI on economic performance of 44 countries over the period 1970-2007. In a cross-country framework, production inefficiencies can be identified as the distance of the individual country's production from the frontier as proxied by the maximum output of the reference country (regarded as an empirical counterpart of an optimal production boundary). Hence, efficiency improvement will represent productivity catch-up via technology diffusion because inefficiencies generally reflect a sluggish adoption of new technologies (Ahn and Sickles 2000).

In this paper we assess the impact of FDI on the production process for small, medium and large countries. We intend to redress an important policy issue of whether the protection-oriented policy will hamper the production efficiency through limiting FDI by explicitly analyzing the relationship between efficiency and openness factor FDI dependent on size of country.

Our findings prove that, especially for medium and big countries, FDI appear to play an important role in accelerating the technological change (shifts in the frontier) but with a decreasing effect at large values of FDI. This result seems to confirm the theoretical hypothesis that FDI leads to increase in productivity by spurring competition: foreign firms have to invest even more in innovation in order to keep up with their technological advantage (Glass and Saggi 1998). The evidence of decreasing returns of FDI can be easily explained by the adjustment costs involved in FDI, e.g. Tybout (1992) and Coe and Helpman (1995). Regarding the effect of FDI on technological catch-up, the evidence reinforces the divergence evolution among countries with respect to FDI (Quah 1996a,b, 1997).

Then, to better analyze the world production technology and have a direct interpretation in terms of elasticity and production technology we follow Florens and Simar (2005) and we project all the pure input-output data points on the robust order-m frontier and we fit Cobb-Douglas parametric frontier model to this cloud of "efficient" points by simple ordinary least squares.

The technology of the world frontier reveals that output is more elastic with respect to labour than capital and the returns to scale are approximately constant. The investigation of the evolution of particular countries at the beginning (1970) and at the end (2007) of observation period, indicates that there is slightly less dispersion of the data and that in both years, the frontier is typically determined by countries with low K/L ratios (US, Germany, Japan). The evidence confirms that the world frontier is at low level of capitalization. Technology change for the frontier countries over this period has been Harrod-neutral (labour-augmenting) technological change.

Finally to assess the diffusion dynamics of world frontier technology with respect to FDI, we explore the evolution over time of the Cobb-Douglas parameters over time for FDI fixed at its three quartiles for each time period.

Our empirical evidence reveals that FDI influence production process through different channels and by enhancing technological changes. We find that FDI has a scale effects on labour by enhancing labour output elasticity but does not influence the output elasticity of capital. FDI is complement with labour through the skills acquired by local labour force as they are trained by the foreign firms; it is substitute with domestic capital goods, this occurs with the adoption of high technology foreign capital goods (Borensztein et al. 1998, De Mello 1999, Xu 2000).

Our results confirm that knowledge embodied in FDI is transferred for technology externalities (shift of the frontier) (Cohen and Levinthal 1989). Hence, our findings support the studies highlighting that lowering barriers to entry of foreign goods and investments have exerted a significantly positive effects on productivity through technology and labour productivity gains, e.g. Borensztein et al. (1998), Cameron et al. (2005).

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	Mean	Std. Dev.	Change (%)
USA	0.761	0.078	-0.077
JPN	0.685	0.055	-0.183
GER	0.680	0.071	0.125
CIV	0.659	0.064	0.259
ISR	0.651	0.097	0.445
FRA	0.650	0.047	0.010
CAN	0.642	0.051	-0.022
ITA	0.637	0.043	-0.049
DOM	0.622	0.068	-0.175
GBR	0.608	0.103	0.428
ESP	0.596	0.041	-0.015
MEX	0.581	0.058	-0.151
HND	0.579	0.117	0.637
NLD	0.572	0.087	0.337
PAN	0.569	0.160	0.091
BOL	0.560	0.120	0.000
IRL	0.544	0.174	-0.506
JAM	0.538	0.143	-0.016
NZL	0.535	0.186	0.919
AUS	0.528	0.073	0.069
MDG	0.525	0.162	0.153
ZWE	0.504	0.093	-0.019
HKG	0.503	0.149	-0.018
BEL	0.498	0.112	0.150
SWE	0.481	0.041	-0.086
ECU	0.480	0.147	-0.043
NOR	0.478	0.166	0.020
ARG	0.471	0.040	-0.046
MWI	0.469	0.139	0.084
AUT	0.466	0.063	-0.274
GRC	0.461	0.099	-0.140
KOR	0.455	0.095	-0.062
KEN	0.452	0.148	0.534
VEN	0.409	0.099	-0.003
DNK	0.402	0.137	-0.032
PRT	0.400	0.045	-0.349
MAR	0.398	0.215	-0.073
FIN	0.387	0.146	-0.022
TUR	0.368	0.043	-0.025
THA	0.363	0.079	0.021
CHL	0.355	0.074	0.043
ZMB	0.344	0.144	-0.357
PHL	0.336	0.033	-0.055
NGA	0.298	0.056	-0.128

Table 1: *Pure efficiency of 44 countries over 1970 till 2007: mean and standard deviation over time and change in % from 1970 to 2007.*

Country	$7-10$ years	$5-7$ years	$3-5$ years
ARG	0.0191	0.0611	0.0011
AUS	0.0264	0.0007	0.0394
AUT	0.1842	0.0003	0.0395
BEL	0.0042	0.0869	0.0019
BOL	0.0042	0.0007	0.0088
CAN	0.0045	0.0350	0.0034
CHL	0.0001	0.1763	0.0020
HKG	0.0001	0.0001	0.0016
CIV	0.1132	0.0006	0.0000
DNK	0.0156	0.0818	0.0003
DOM	0.0144	0.0008	0.0004
ECU	0.0167	0.0007	0.0326
FIN	0.0030	0.1622	0.0017
FRA	0.0049	0.0008	0.0361
GER	0.0108	0.0007	0.0033
GRC	0.0347	0.0007	0.0011
HND	0.0295	0.0004	0.1649
IRL	0.0566	0.0030	0.0074
ISR	0.0080	0.0190	0.0014
ITA	0.0235	0.0001	0.0274
JAM	0.0568	0.0047	0.0077
JPN	0.0132	0.0231	0.0157
KEN	0.0060	0.0002	0.3042
KOR	0.0011	0.0055	0.0243
MDG	0.0063	0.0811	0.0155
MW I	0.0015	0.0003	0.0154
MEX	0.0040	0.0010	0.0424
MAR	0.1004	0.0005	0.0455
NLD	0.0692	0.0094	0.1219
NZL	0.0780	0.0057	0.0301
NGA	0.0009	0.0023	0.0849
NOR	0.0108	0.0021	0.0922
PAN	0.2324	0.0020	0.0402
PHL	0.0110	0.0007	0.1333
PRT	0.0041	0.0011	0.2447
ESP	0.1493	0.0025	0.0048
SWE	0.0087	0.0287	0.0031
THA	0.0076	0.0035	0.0012
TUR	0.0162	0.0022	0.0196
GBR	0.0001	0.0065	0.0960
USA	0.0007	0.0078	0.2496
VEN	0.0032	0.0108	0.0015
ZMB	0.0235	0.0103	0.0044
ZWE	0.0148	0.0018	0.0324

Table 2: *Spectral analysis of pure efficiency. The columns on share of total variance report the estimated efficiency variance shares over each frequency range.*

Estimator	α	β_1	β_2	R^2
order-m	0.3766	0.3145	0.7199	0.89
FDH	0.4335	0.3359	0.7335	0.89
COLS	-0.0084	0.2337	0.8133	0.95

Table 3: *Estimates of the parameter of the best Cobb-Douglas approximation of the world frontier. The R^2 for order-m and FDH indicate the quality of the approximation. For the COLS, the R^2 is from the OLS estimation of the average production function.*

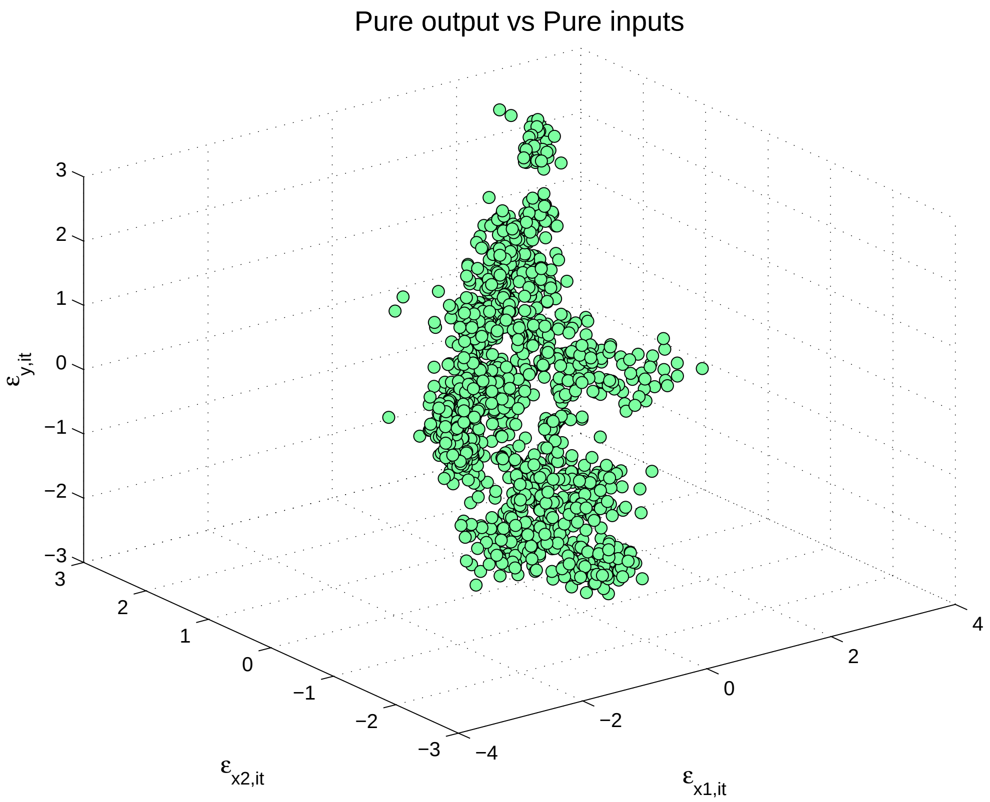


Figure 1: *Estimated "pure" output $\widehat{\varepsilon}_{y,it}$ (GDP) and inputs $\widehat{\varepsilon}_{x1,it}$ (Capital) and $\widehat{\varepsilon}_{x2,it}$ (Labour).*

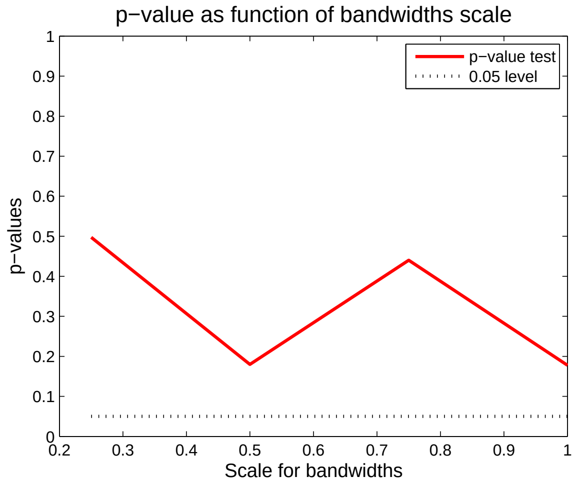


Figure 2: *Test of independence between* $(\varepsilon_x, \varepsilon_y)$ *and* (F_t, Z)*. Resulting p-values for selected scaling factor c of the optimal bandwidths.*

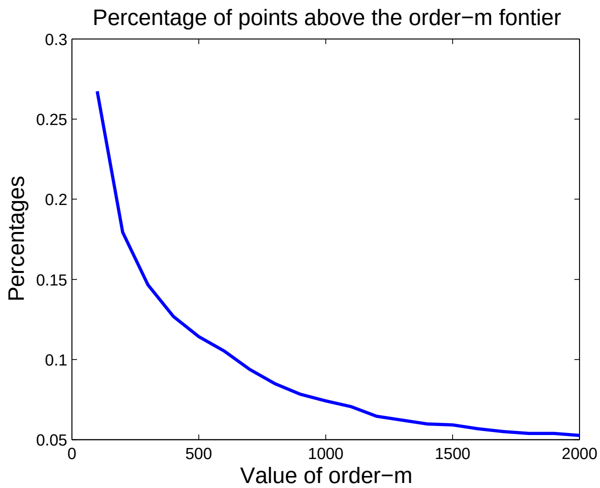


Figure 3: *Percent of points outside the m-frontier at each value of* m*.*

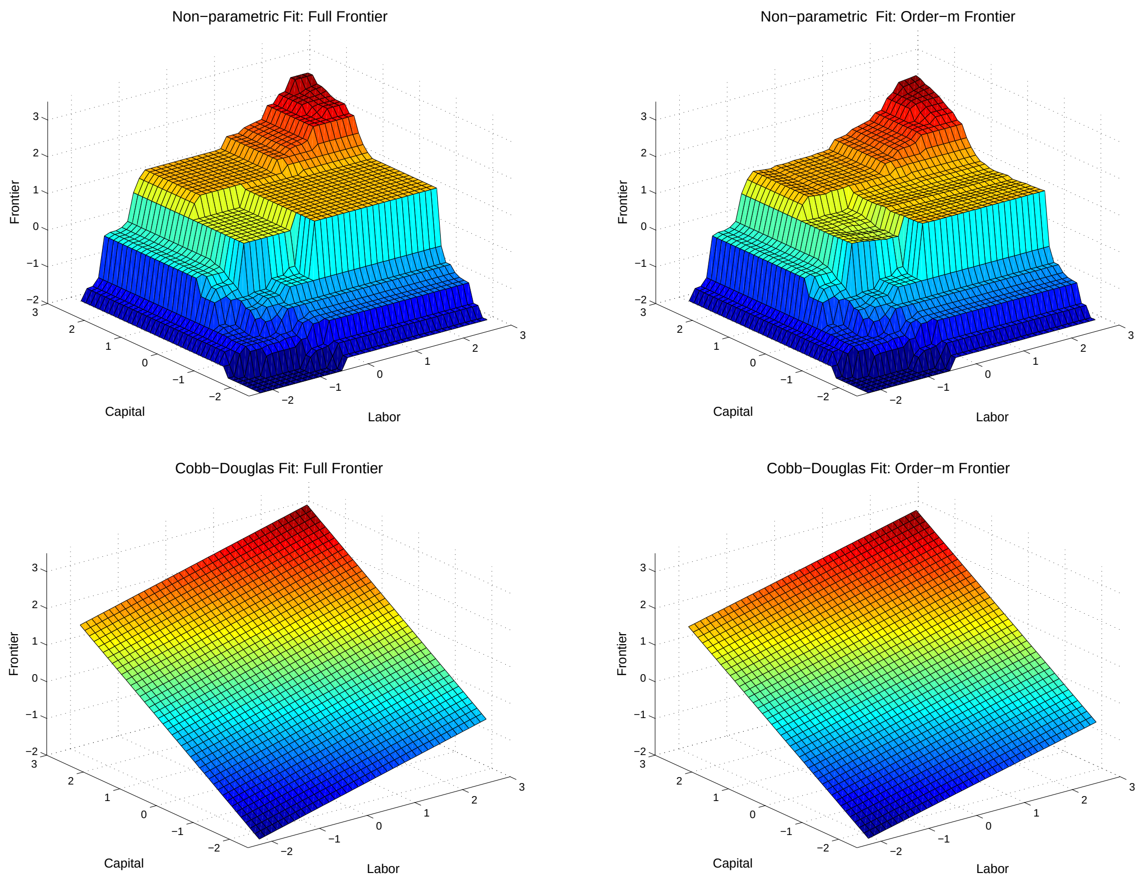


Figure 4: *The first two top panels represent the non-parametric frontiers in "pure" units, left panel is the full FDH and right panel is the order* $-m$ *frontier estimate. The two bottom panels are their respective Cobb-Douglas approximations.*

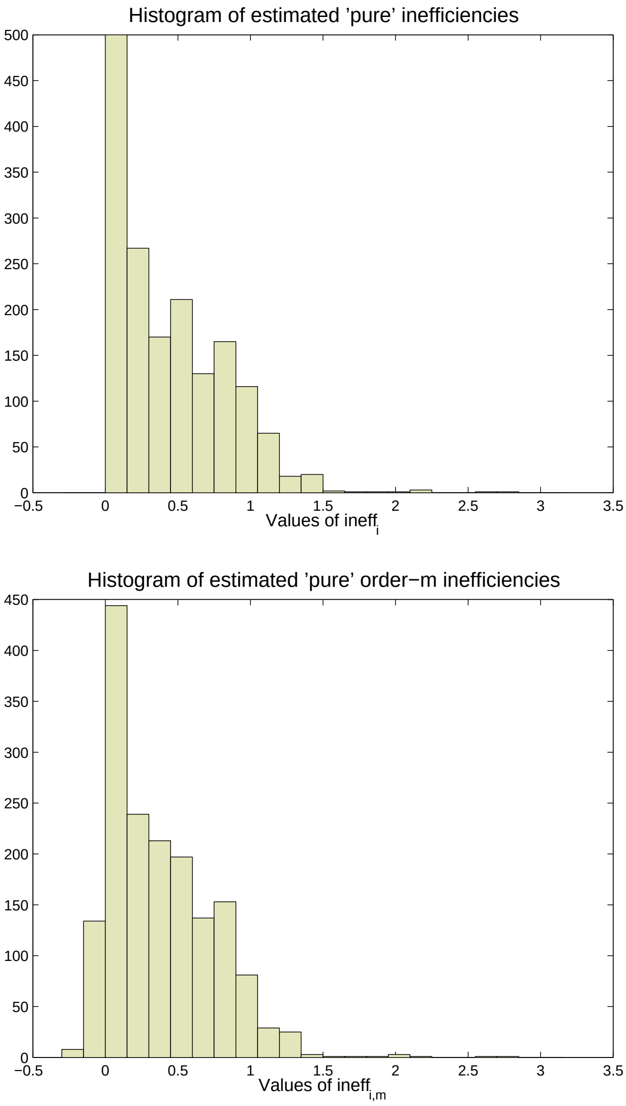
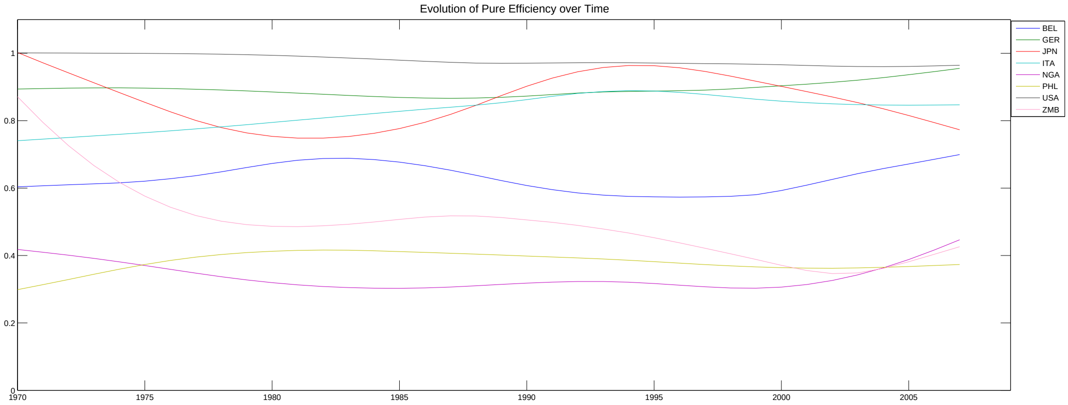


Figure 5: *Distribution of the estimated inefficiencies, relative to the full frontier* $\widehat{\varphi}$ *(top panel) and to the order-m frontier* $\widehat{\varphi}_m$ *(bottom panel).*



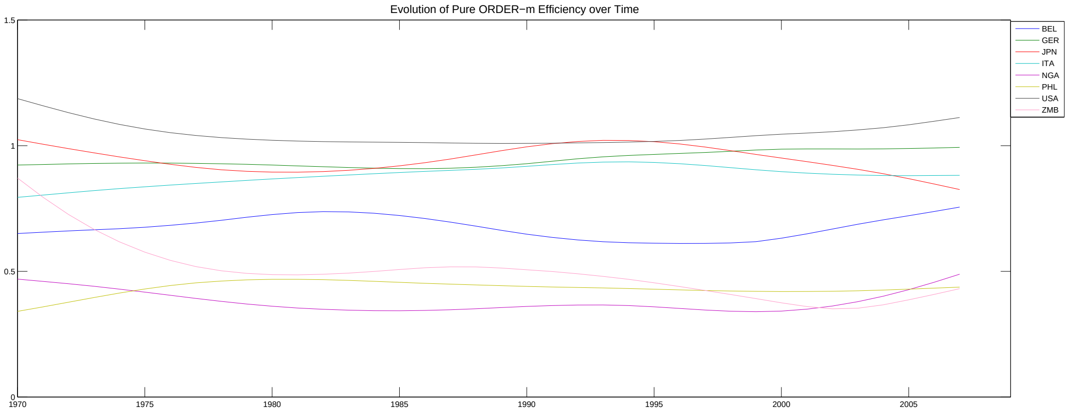


Figure 6: *Time-varying pure efficiency for Belgium (BEL), Germany (GER), Italy (ITA), Japan (JPN), Nicaragua (NGA), Philippines (PHL), United States (USA) and Zambia (ZMB).*

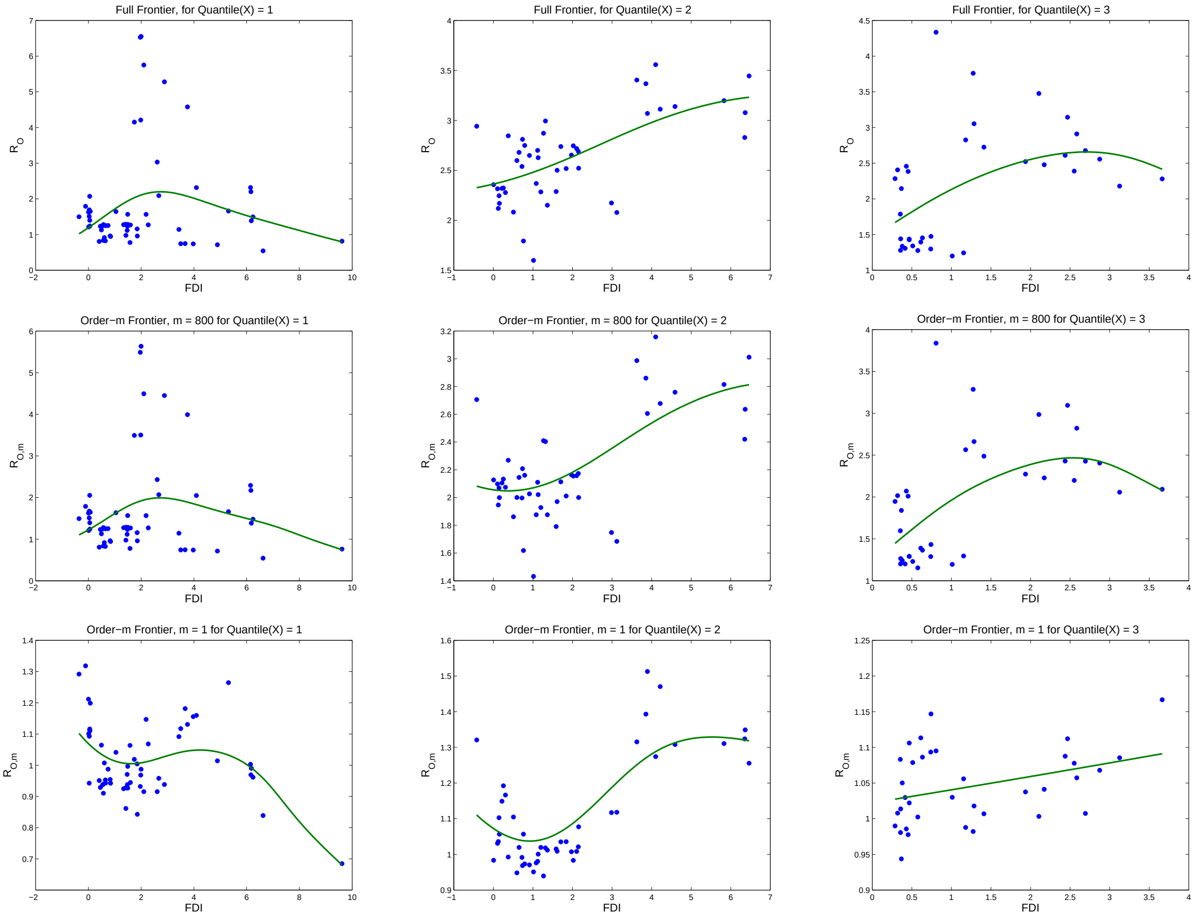


Figure 7: *The first three top panels represent the full ratios* $\widehat{R}_O(x, y|z, f_t)$ *as a function of FDI with the two inputs (labour and capital) fixed at their three quartiles, from left to right,* $Q_{0.25}, Q_{0.50}, Q_{0.75}$*; the middle panels are the corresponding order-m ratios* $\widehat{R}_{O,m}(x, y|z, f_t)$ *for* $m = 800$*, the bottom panels are for* $m = 1$*.*

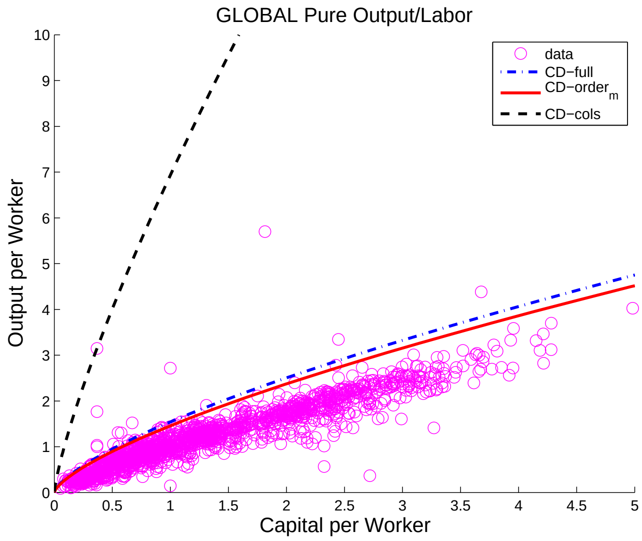


Figure 8: *Global Cobb-Douglas full and order* $-m$ *and COLS frontiers in pure output per labour and capital per labour units.*

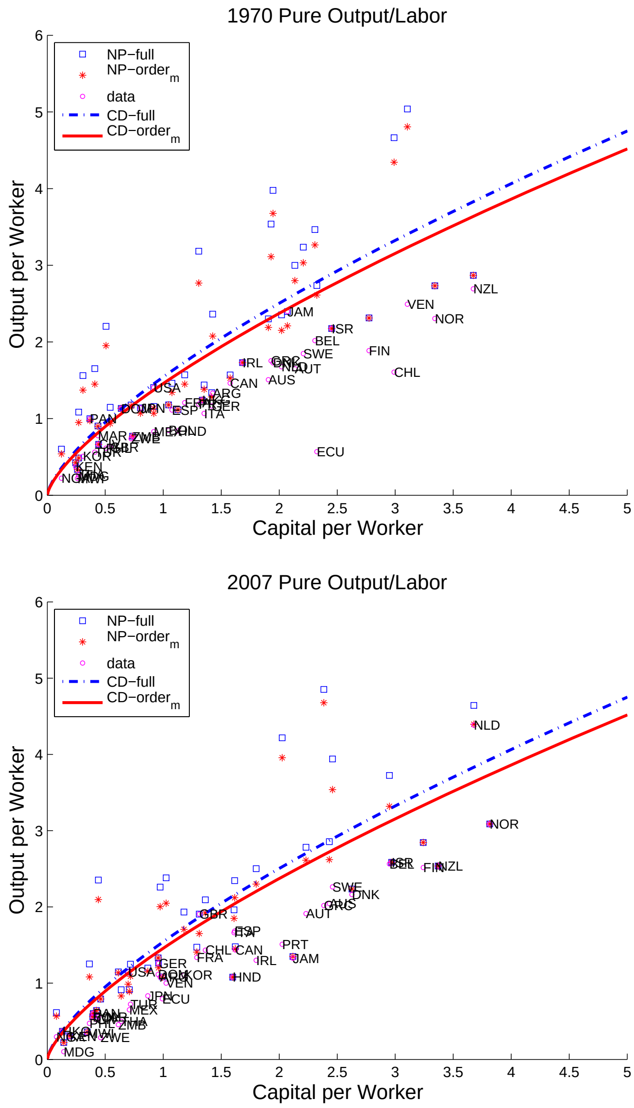


Figure 9: *1970 and 2007 Non-parametric and Cobb-Douglas full and order $-$ m frontiers in pure output per labour and capital per labour units, (top panels); global frontier in pure output per labour and capital per labour units (bottom panel).*

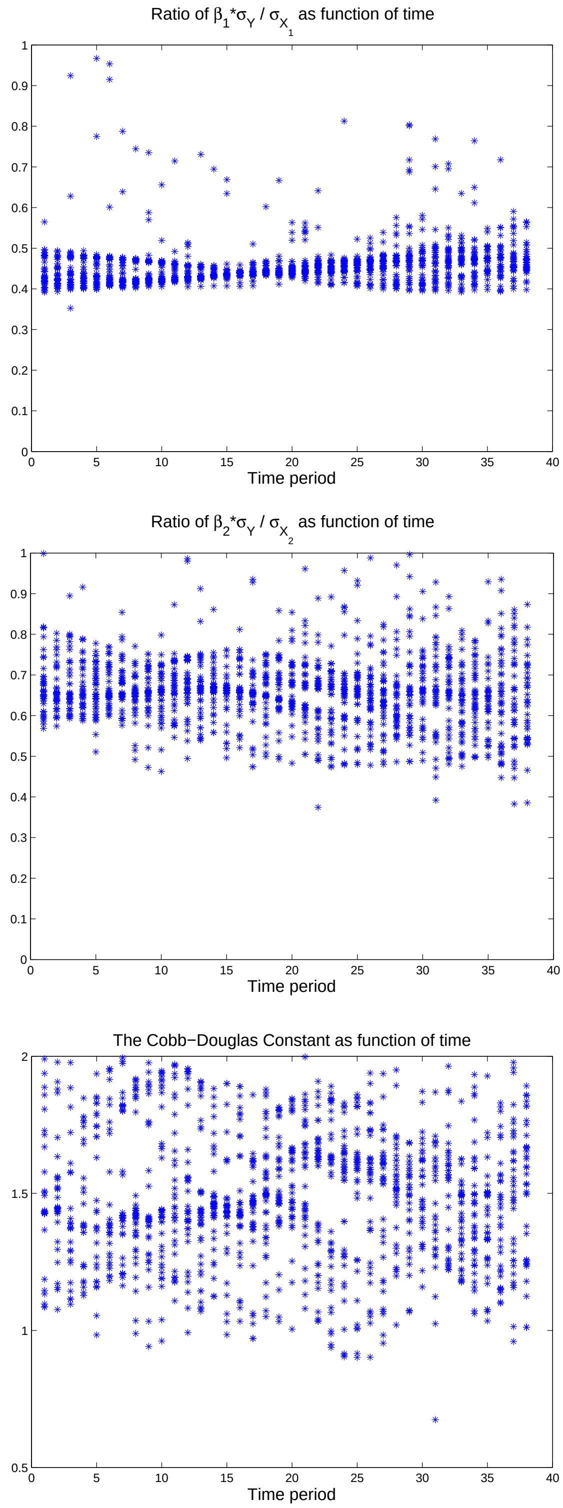


Figure 10: *Parameters estimates of Cobb- Douglas World Production frontier: output elasticities of capital and labour (top panels) and technological shift captured by the constant parameter (bottom panel).*

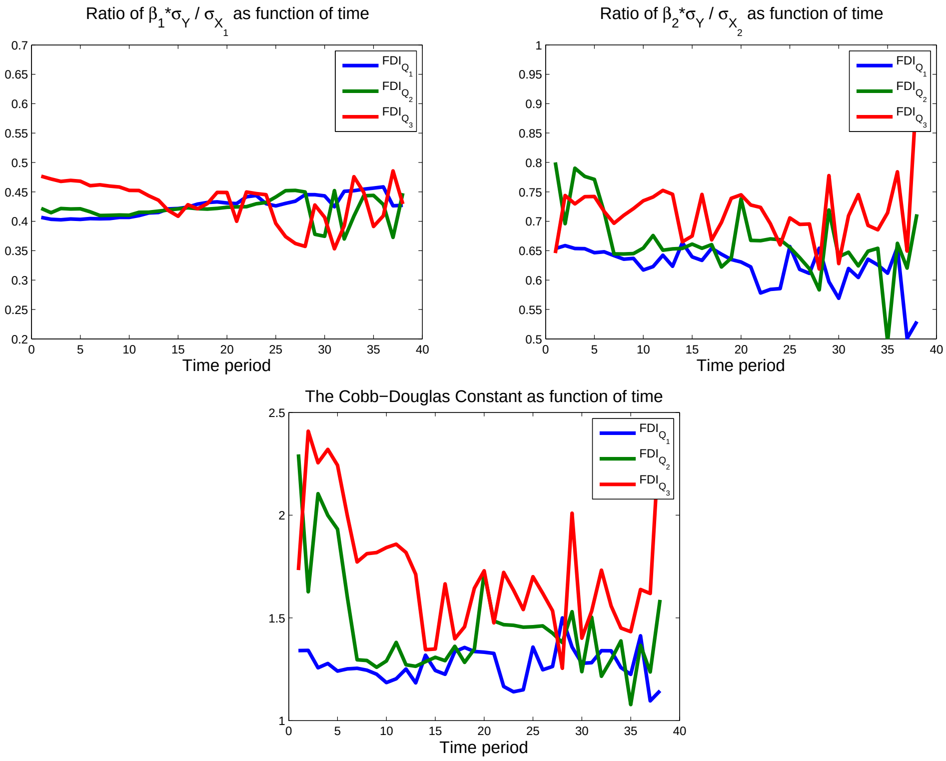


Figure 11: *Parameters estimates of Cobb- Douglas World Production frontier when fixing the level of* $Z = FDI$ *and* f_t*. Here* f_t *is fixed at its median value, and* FDI *is fixed at its 3 quartiles. Output elasticities of capital and labour (top panels) and technological shift captured by the constant parameter (bottom panel).*