TESTING AND CORRECTING FOR ENDOGENEITY IN NONLINEAR UNOBSERVED EFFECTS MODELS

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1. Introduction

- In unobserved effects models we can think of two kinds of endogeneity for an explanatory variable:

1. Correlation with unobserved effect(s) (time constant)
2. Correlation with innovations (time varying)

- Application to linear model: Levitt (1996, QJE), effects of prison size on violent crime.
- Nonlinear models: Can combine the correlated random effects (CRE) and control function (CF) approaches for certain nonlinear models.

- Papke and Wooldridge (2008, J of E) provides one approach. Simple but not ideal for testing purposes: It conflates the two kinds of endogeneity.
- Application to effects of spending on school/district test pass rates.
- Here focus on continuous endogenous explanatory variables (EEVs), but some suggestions for discrete EEVs.

- Other Approaches:

1. “Fixed Effects” (Heterogeneity as parameters to estimate):

    - Incidental parameters problem with small T.
    - Bias adjustments available for parameters and average partial effects, but usually stationarity and weak dependence (even independence) are assumed.
    - Difficult to incorporate time effects.
    - Endogenous explanatory variables?

2. Conditional MLE:

- Only works in special cases.
- Relies on conditional independence across time.
- Partial effects in nonlinear models often unidentified.
- Exentensions to EEVs?

3. Finite Number of Types (Bonhomme and Manresa)

- Conditional Independence
- Nonlinear Models?
- Extension to EEVs?

	Ideal	FE	CMLE	CRE
Restricts $D(c_i\|x_i)$?	No	No	No	Yes
Incidental Parameters with Small T?	No	Yes	No	No
Restricts Time Series Dependence/Heterogeneity?	No	Yes (1)	Yes (2)	No
Restricts Amount of Heterogeneity?	No	No (3)	Yes	No
APEs Identified?	Yes	Yes (4)	No	Yes
Unbalanced Panels?	Yes	Yes	Yes	Yes (5)
Can Estimate $D(c_i)$?	Yes	Yes (4)	No	Yes (6)
Endogenous Explanatory Variables?	Yes	Yes (4)	No (7)	Yes

1. The large T approximations assume weak dependence and often stationarity.

2. Usually conditional independence, unless estimator is inherently fully robust (linear, Poisson).

3. Need at least one more time period than sources of heterogeneity.

4. Subject to the incidental parameters problem.

5. Subject to exchangeability restrictions.

6. Under conditional independence or some other restriction.

7. Unless one makes parametric assumptions on the reduced form and imposes conditional independence.

2. Linear Model

- Consider a "structural" equation

$$y_{it1} = \mathbf{x}_{it1}\boldsymbol{\beta}_1 + c_{i1} + u_{it1}$$

where

$$\mathbf{x}_{it1} = (\mathbf{z}_{it1}, \mathbf{y}_{it2})$$

- The outside instruments are $\mathbf{z}_{it2}$.
- $\mathbf{z}_{it1}$ can include time effects, but supress.

- Both $\{\mathbf{z}_{it}\}$ and $\{\mathbf{y}_{it2}\}$ may be correlated with c_{i1}.
- Assume $\{\mathbf{z}_{it}\}$ is strictly exogenous with respect to $\{u_{it1}\}$:

$$Cov(\mathbf{z}_{it}, u_{ir1}) = 0, \text{ all } t, r = 1, \ldots, T$$

- $\{\mathbf{y}_{it2}\}$ may be correlated with $\{u_{it1}\}$ (across all time periods).
- Given a rank condition, $\boldsymbol{\beta}_1$ can be estimated by fixed effects IV (FEIV).

- How can we test the null hypothesis that $\{\mathbf{y}_{it2}\}$ is exogenous with respect to $\{u_{it1}\}$?
- Hausman test comparing FE and FEIV.
  - Cumbersome due to deficient rank.
  - Original Hausman test not robust to serial correlation or heteroskedasticity in $\{u_{it1}\}$.

- Variable Addition Test (Control Function):

1. Estimate the reduced form of $\mathbf{y}_{it2}$,

$$\mathbf{y}_{it2} = \mathbf{z}_{it}\mathbf{\Pi}_2 + \mathbf{c}_{i2} + \mathbf{u}_{it2},$$

by fixed effects, and obtain the FE residuals,

$$\widehat{\ddot{\mathbf{u}}}_{it2} = \ddot{\mathbf{y}}_{it2} - \ddot{\mathbf{z}}_{it}\hat{\mathbf{\Pi}}_2$$

$$\ddot{\mathbf{y}}_{it2} = \mathbf{y}_{it2} - T^{-1}\sum_{r=1}^{T}\mathbf{y}_{ir2}$$

2. Estimate the equation

$$y_{it1} = \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \hat{\ddot{\mathbf{u}}}_{it2}\boldsymbol{\rho}_1 + c_{i1} + error_{it1}$$

by usual FE and compute a robust test of $H_0 : \boldsymbol{\rho}_1 = \mathbf{0}$.

- In step (2), $\hat{\boldsymbol{\beta}}_1$ is the FEIV estimator.
- Note that the nature of $\mathbf{y}_{it2}$ is unrestricted (discrete, continuous, both features).

- We can also use a correlated random effects approach, but some but some care is needed to get a proper test.
- The Mundlak equation for $\mathbf{y}_{it2}$ is

$$\mathbf{y}_{it2} = \boldsymbol{\psi}_2 + \mathbf{z}_{it}\boldsymbol{\Pi}_2 + \bar{\mathbf{z}}_i\boldsymbol{\Xi}_2 + \mathbf{a}_{i2} + \mathbf{u}_{it2}$$

$$\bar{\mathbf{z}}_i = T^{-1}\sum_{t=1}^{T}\mathbf{z}_{it}$$

- We are operating *as if*

$$Cov(\mathbf{z}_{it}, \mathbf{u}_{is2}) = \mathbf{0}, \text{ all } t, s$$

$$Cov(\mathbf{z}_{it}, \mathbf{a}_{i2}) = \mathbf{0}, \text{ all } t$$

- Key: How should we apply the Mundlak device to c_{i1} in

$$y_{it1} = \mathbf{x}_{it1}\boldsymbol{\beta}_1 + c_{i1} + u_{it1}?$$

- Projecting c_{i1} only onto $\bar{\mathbf{z}}_i$ is fine for estimation.
- For testing, it does not distinguish between

$$Cov(\mathbf{y}_{it2}, c_{i1}) \neq \mathbf{0}$$

and

$$Cov(\mathbf{y}_{it2}, u_{is1}) \neq \mathbf{0}.$$

- Better is to project c_{i1} onto $(\bar{\mathbf{z}}_i, \bar{\mathbf{v}}_{i2})$ where

$$\mathbf{y}_{it2} = \boldsymbol{\psi}_2 + \mathbf{z}_{it}\boldsymbol{\Pi}_2 + \bar{\mathbf{z}}_i\boldsymbol{\Xi}_2 + \mathbf{v}_{it2}$$

$$\begin{aligned} c_{i1} &= \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\lambda}_1 + \bar{\mathbf{v}}_{i2}\boldsymbol{\pi}_1 + a_{i1} \\ Cov(\mathbf{z}_i, a_{i1}) &= \mathbf{0} \\ Cov(\mathbf{y}_{i2}, a_{i1}) &= \mathbf{0} \end{aligned}$$

- Plugging in gives the estimating equation

$$\begin{aligned} y_{it1} &= \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\lambda}_1 + \bar{\mathbf{v}}_{i2}\boldsymbol{\pi}_1 + a_{i1} + u_{it1} \\ &= \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + a_{i1} + u_{it1} \end{aligned}$$

- By the Mundlak device, a_{i1} is uncorrelated with all RHS observables.
- By assumption, $\mathbf{z}_i$ is uncorrelated with u_{it1}.
- Now test whether $\mathbf{y}_{it2}$, equivalently $\mathbf{v}_{it2}$, is uncorrelated with u_{it1}.

1. Run a pooled OLS regression (or use random effects),

$$\mathbf{y}_{it2} = \boldsymbol{\psi}_2 + \mathbf{z}_{it}\boldsymbol{\Pi}_2 + \bar{\mathbf{z}}_i\boldsymbol{\Xi}_2 + \mathbf{v}_{it2},$$

and obtain the residuals, $\hat{\mathbf{v}}_{it2}$.

2. Estimate

$$y_{it1} = \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + \hat{\mathbf{v}}_{it2}\boldsymbol{\rho}_1 + error_{it1}$$

by POLS or RE.

3. Use a robust Wald test of $H_0 : \boldsymbol{\rho}_1 = \mathbf{0}$.

- Algebra:

(i) $\hat{\boldsymbol{\beta}}_1$ in step (2) is the FEIV estimator.

(ii) $\hat{\boldsymbol{\rho}}_1$ is identical to that from estimating

$$y_{it1} = \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \widehat{\ddot{\mathbf{u}}}_{it2}\boldsymbol{\rho}_1 + c_{i1} + error_{it1}$$

by FE.

- Result (i) still holds if $\bar{\mathbf{y}}_{i2}$ is dropped from the estimating equation, but (ii) does not.

- Conclusion: In using the CRE/CF approach for testing

$$H_0 : Cov(\mathbf{y}_{it2}, u_{is1}) = \mathbf{0},$$

use the equations

$$\mathbf{y}_{it2} = \hat{\boldsymbol{\psi}}_2 + \mathbf{z}_{it}\hat{\boldsymbol{\Pi}}_2 + \bar{\mathbf{z}}_i\hat{\boldsymbol{\Xi}}_2 + \hat{\mathbf{v}}_{it2}$$

$$y_{it1} = \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + \hat{\mathbf{v}}_{it2}\boldsymbol{\rho}_1 + error_{it1}$$

- Also works in the unbalanced case when the complete cases are used (Joshi and Wooldridge, 2015).

- What about using Chamberlain in place of Mundlak?
- Reusing notation, with

$$\mathbf{z}_i = (\mathbf{z}_{i1}, \ldots, \mathbf{z}_{iT}), \mathbf{y}_{i2} = (\mathbf{y}_{i12}, \ldots, \mathbf{y}_{iT2}),$$

$$\mathbf{y}_{it2} = \hat{\boldsymbol{\psi}}_2 + \mathbf{z}_{it}\hat{\boldsymbol{\Pi}}_2 + \mathbf{z}_i\hat{\boldsymbol{\Xi}}_2 + \hat{\mathbf{v}}_{it2}$$

$$y_{it1} = \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \psi_1 + \mathbf{z}_i\boldsymbol{\xi}_1 + \mathbf{y}_{i2}\boldsymbol{\pi}_1 + \hat{\mathbf{v}}_{it2}\boldsymbol{\rho}_1 + error_{it1}$$

- Estimates of $\boldsymbol{\beta}_1$ and $\boldsymbol{\rho}_1$ are are identical to Mundlak, as is robust Wald test. (POLS or RE).

- Conclusion: Include time averages of $\{\mathbf{z}_{it}\}$ and $\{\mathbf{y}_{it2}\}$ to obtain a clean test of endogeneity of $\{\mathbf{y}_{it2}\}$ with respect to $\{u_{it1}\}$.
- If POLS or RE are used, Chamberlain = Mundlak.
- All goes through with time effects.
- Time constant variables can be included in the CRE/CF approach.

- Ignoring the pre-testing problem, a strategy for testing is:

1. If the VAT rejects, use FEIV.

    ▸ Or, then test REIV against FEIV, as instruments may be “super” exogenous.

2. If the VAT fails to reject, use FE or compare RE and FE.

3. Exponential Model

- Fully robust test for exogeneity:

1. Estimate the reduced form for $\mathbf{y}_{it2}$ by fixed effects and obtain the FE residuals,

$$\hat{\ddot{\mathbf{u}}}_{it2} = \ddot{\mathbf{y}}_{it2} - \ddot{\mathbf{z}}_{it}\hat{\mathbf{\Pi}}_2$$

2. Use FE Poisson on the mean function

$$\text{“}E(y_{it1}|\mathbf{z}_{it1}, \mathbf{y}_{it2}, \hat{\ddot{\mathbf{u}}}_{it2}, c_{i1}) = c_{i1}\exp(\mathbf{x}_{it1}\boldsymbol{\beta}_1 + \hat{\ddot{\mathbf{u}}}_{it2}\boldsymbol{\rho}_1)\text{”}$$

and use a robust Wald test of $H_0 : \boldsymbol{\rho}_1 = \mathbf{0}$.

- The null hypothesis is

$$H_0 \ : \ E(y_{it1}|\mathbf{z}_{i1},\mathbf{y}_{i2},\ddot{\mathbf{u}}_{it2},c_{i1}) = E(y_{it1}|\mathbf{z}_{it1},\mathbf{y}_{it2},c_{i1})$$
$$= c_{i1}\exp(\mathbf{x}_{it1}\boldsymbol{\beta}_1)$$

- Algebra: The Poisson FE estimates of $(\boldsymbol{\beta}_1,\boldsymbol{\rho}_1)$ are unchanged if we estimate the Mundlak reduced form:

$$\mathbf{y}_{it2} = \boldsymbol{\psi}_2 + \mathbf{z}_{it}\boldsymbol{\Pi}_2 + \bar{\mathbf{z}}_i\boldsymbol{\Xi}_2 + \mathbf{v}_{it2}$$

and use residuals, $\hat{\mathbf{v}}_{it2}$ ($\hat{\mathbf{v}}_{it2} \neq \widehat{\ddot{\mathbf{u}}}_{it2}$ but $\widehat{\ddot{\mathbf{v}}}_{it2} = \widehat{\ddot{\mathbf{u}}}_{it2}$).

- For testing, no restrictions are put on the RF of $\mathbf{y}_{it2}$.
- When would the Mundlak/FE Poisson approach be consistent for $\boldsymbol{\beta}_1$?
- Assume that

$$\begin{aligned} E(y_{it1}|\mathbf{z}_i, \mathbf{y}_{i2}, c_{i1}, u_{it1}) &= E(y_{it1}|\mathbf{z}_{it1}, \mathbf{y}_{it2}, c_{i1}, u_{it1}) \\ &= c_{i1}\exp(\mathbf{x}_{it1}\boldsymbol{\beta}_1 + u_{it1}) \end{aligned}$$

where $\mathbf{x}_{it1}$ can be any function of $(\mathbf{z}_{it1}, \mathbf{y}_{it2})$.

- The reduced form is

$$\begin{aligned}\mathbf{y}_{it2} &= \boldsymbol{\psi}_2 + \mathbf{z}_{it}\boldsymbol{\Pi}_2 + \bar{\mathbf{z}}_i\boldsymbol{\Xi}_2 + \mathbf{a}_{i2} + \mathbf{u}_{it2} \\ \mathbf{v}_{it2} &= \mathbf{a}_{i2} + \mathbf{u}_{it2}\end{aligned}$$

- Sufficient is

$$\begin{aligned}u_{it1} &= \mathbf{u}_{it2}\boldsymbol{\rho}_1 + e_{it1} \\ &= \mathbf{v}_{it2}\boldsymbol{\rho}_1 - \mathbf{a}_{i2}\boldsymbol{\rho}_1 + e_{it1} \\ & e_{it1} \perp (\mathbf{z}_i, c_{i1}, \mathbf{c}_{i2}, \mathbf{u}_{i2})\end{aligned}$$

- This appears to impose a restriction of only contemporaneous correlation between $\{u_{it1}\}$ and $\{\mathbf{u}_{it2}\}$. How important is it?
- In the linear case it makes no difference.

- Is consistently estimating $\boldsymbol{\beta}_1$ enough?
- The average structural function (Blundell and Powell, 2003) is identified:

$$
\begin{aligned}
ASF(\mathbf{x}_{t1}) &= E_{(c_{i1},u_{it1})}[c_{i1}\exp(\mathbf{x}_{t1}\boldsymbol{\beta}_1 + u_{it1})] \\
&= E_{(c_{i1},u_{it1})}[c_{i1}\exp(u_{it1})]\exp(\mathbf{x}_{t1}\boldsymbol{\beta}_1)
\end{aligned}
$$

- But the average partial effects are not generally identified.
- For a continuous x_{t1j},

$$
APE_{tj} = \beta_{1j}E_{(\mathbf{x}_{it1},c_{i1},u_{it1})}[c_{i1}\exp(\mathbf{x}_{it1}\boldsymbol{\beta}_1 + u_{it1})]
$$

• A CRE/CF approach identifies both. What could it look like?

• At least two choices. Let

$$
\begin{aligned}
E(y_{it1}|\mathbf{z}_{it1}, \mathbf{y}_{it2}, c_{i1}, u_{it1}) &= c_{i1}\exp(\mathbf{x}_{it1}\boldsymbol{\beta}_1 + u_{it1}) \\
v_{it1} &= c_{i1}\exp(u_{it1}) \\
\mathbf{y}_{it2} &= \boldsymbol{\psi}_2 + \mathbf{z}_{it}\boldsymbol{\Pi}_2 + \bar{\mathbf{z}}_i\boldsymbol{\Xi}_2 + \mathbf{v}_{it2}
\end{aligned}
$$

1. Use Mundlak (or Chamberlain) on

$$
D(v_{it1}|\mathbf{z}_i, \mathbf{v}_{it2}) = D(v_{it1}|\bar{\mathbf{z}}_i, \mathbf{v}_{it2})
$$

(Papke and Wooldridge, 2008).

- Uses weaker exogeneity requirements (but not in linear model).
- Cannot use GLS-type methods; pooled methods or GMM from moment conditions.
- Does not lead to cleanest test of endogeneity.

2. Use Mundlak (or Chamberlain) on

$$\begin{aligned} D(v_{it1}|\mathbf{z}_i, \mathbf{v}_{i2}) &= D(v_{it1}|\bar{\mathbf{z}}_i, \bar{\mathbf{v}}_{i2}, \mathbf{v}_{it2}) \\ &= D(v_{it1}|\bar{\mathbf{z}}_i, \bar{\mathbf{y}}_{i2}, \mathbf{v}_{it2}) \end{aligned}$$

- Strict exogeneity is assumed, so GLS-type procedures can be used.
- Separates endogeneity of $\{\mathbf{y}_{it2}\}$ with respect to c_{i1} and $\{u_{it1}\}$.
- Because of the linear index structure, we can use the Mundlak residuals or FE residuals for the RF of $\mathbf{y}_{it2}$.

- In the simplest case, the estimating equation is

$$E(y_{it1}|\mathbf{z}_i, \mathbf{y}_{i2}) = \exp(\psi_1 + \mathbf{x}_{it1}\boldsymbol{\beta}_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + \mathbf{v}_{it2}\boldsymbol{\rho}_1)$$

and the Mundlak or FE residuals are plugged in for $\mathbf{v}_{it2}$.

- One can apply GLS ("generalized estimating equations) methods, or GMM.
- Unlike in the linear case, no equivalance between CRE and FE Poisson estimates.
- Wooldridge (1991)/Windmeijer (2000) moments approach is an alternative.

4. Probit Response Function

- Now consider a probit conditional mean for $y_{it1} \in [0,1]$:

$$E(y_{it1}|\mathbf{z}_i, \mathbf{y}_{i2}, r_{it1}) = E(y_{it1}|\mathbf{z}_{it1}, \mathbf{y}_{it2}, r_{it1}) = \Phi(\mathbf{x}_{it1}\boldsymbol{\beta}_1 + r_{it1})$$

- Thinking of

$$r_{it1} = c_{i1} + u_{it1}$$

- For continuous EEVs,

$$\mathbf{y}_{it2} = \boldsymbol{\psi}_2 + \mathbf{z}_{it}\boldsymbol{\Pi}_2 + \bar{\mathbf{z}}_i\boldsymbol{\Xi}_2 + \mathbf{v}_{it2}$$

- Assume

$$\mathbf{v}_{it2} \text{ is independent of } \mathbf{z}_i.$$

- Key assumption:

$$\begin{aligned} D(r_{it1}|\mathbf{z}_i, \mathbf{v}_{i2}) &= D(r_{it1}|\bar{\mathbf{z}}_i, \bar{\mathbf{v}}_{i2}, \mathbf{v}_{it2}) \\ &= D(r_{it1}|\bar{\mathbf{z}}_i, \bar{\mathbf{y}}_{i2}, \mathbf{v}_{it2}) \end{aligned}$$

- Leading case: homoskedastic normal with linear mean:

$$r_{it1}|\bar{\mathbf{z}}_i, \bar{\mathbf{y}}_{i2}, \mathbf{v}_{it2} \sim Normal(\psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + \mathbf{v}_{it2}\boldsymbol{\rho}_1, 1)$$

(Variance normalization has no effect on ASF or APEs.)

• Then

$$E(y_{it1}|\mathbf{z}_i, \mathbf{y}_{i2}) = \Phi(\mathbf{x}_{it1}\boldsymbol{\beta}_1 + \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + \mathbf{v}_{it2}\boldsymbol{\rho}_1)$$

and two-step procedures are immediate:

1. Obtain $\hat{\mathbf{v}}_{it2}$ by pooled OLS.

2. Insert $\hat{\mathbf{v}}_{it2}$ in place of $\mathbf{v}_{it2}$, use pooled (fractional) probit.

• Can use a quasi-GLS procedure (GEE) because of strict exogeneity.

• Can test $H_0 : \boldsymbol{\rho}_1 = \mathbf{0}$ using a robust Wald test.

- ASF is identified from

$$ASF(\mathbf{x}_{t1}) = E_{(\bar{\mathbf{z}}_i,\bar{\mathbf{y}}_{i2},\mathbf{v}_{it2})}[\Phi(\mathbf{x}_{t1}\boldsymbol{\beta}_1 + \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + \mathbf{v}_{it2}\boldsymbol{\rho}_1)]$$

$$\widehat{ASF}(\mathbf{x}_{t1}) = N^{-1}\sum_{t=1}^{T}\Big[\Phi(\mathbf{x}_{t1}\hat{\boldsymbol{\beta}}_1 + \hat{\psi}_1 + \bar{\mathbf{z}}_i\hat{\boldsymbol{\xi}}_1 + \bar{\mathbf{y}}_{i2}\hat{\boldsymbol{\pi}}_1 + \hat{\mathbf{v}}_{it2}\hat{\boldsymbol{\rho}}_1)\Big]$$

- APEs:

$$\beta_{1j}E_{(\mathbf{x}_{it1},\bar{\mathbf{z}}_i,\bar{\mathbf{y}}_{i2},\mathbf{v}_{it2})}[\phi(\mathbf{x}_{it1}\boldsymbol{\beta}_1 + \psi_1 + \bar{\mathbf{z}}_i\boldsymbol{\xi}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\pi}_1 + \mathbf{v}_{it2}\boldsymbol{\rho}_1)]$$

- Stata `margins` command gets the correct estimates, but does not adjust inference for two-step estimation.

- Lots of useful embellishments. For example,

$$Var(r_{it1}|\bar{\mathbf{z}}_i, \bar{\mathbf{y}}_{i2}, \mathbf{v}_{it2}) = \exp[2(\bar{\mathbf{z}}_i\boldsymbol{\theta}_1 + \bar{\mathbf{y}}_{i2}\boldsymbol{\gamma}_1 + \mathbf{v}_{it2}\boldsymbol{\kappa}_1)]$$

- Run a heteroskedastic "probit" with mean function depending on

$$(1, \mathbf{x}_{it1}, \bar{\mathbf{z}}_i, \bar{\mathbf{y}}_{i2}, \hat{\mathbf{v}}_{it2})$$

and variance function depending on

$$(\bar{\mathbf{z}}_i, \bar{\mathbf{y}}_{i2}, \hat{\mathbf{v}}_{it2}).$$

- The ASF is consistently estimated as

$$\widehat{ASF}(\mathbf{x}_{t1}) = N^{-1}\sum_{t=1}^{T}\Phi\left(\frac{\mathbf{x}_{t1}\hat{\boldsymbol{\beta}}_1 + \hat{\psi}_1 + \bar{\mathbf{z}}_i\hat{\boldsymbol{\xi}}_1 + \bar{\mathbf{y}}_{i2}\hat{\boldsymbol{\pi}}_1 + \hat{\mathbf{v}}_{it2}\hat{\boldsymbol{\rho}}_1}{\exp\left(\bar{\mathbf{z}}_i\hat{\boldsymbol{\theta}}_1 + \bar{\mathbf{y}}_{i2}\hat{\boldsymbol{\gamma}}_1 + \hat{\mathbf{v}}_{it2}\hat{\boldsymbol{\kappa}}_1\right)}\right)$$

- Stata 14 does the estimation with `fracreg` (pooled estimation).

- Stata `margins` (should) give the correct estimates for the APEs because $\mathbf{x}_{it1}$ appears only in the "mean" function.

- In the spirit of Blundell and Powell (2004, REStud), one can directly use flexible functional forms (squares, interactions, higher order terms) in

$$(\mathbf{x}_{it1}, \bar{\mathbf{z}}_i, \bar{\mathbf{y}}_{i2}, \hat{\mathbf{v}}_{it2})$$

and compute partial effects with respect to $\mathbf{x}_{it1}$. Average out the other variables.

- As in Altonji and Matzkin (2005, Econometrica), functions other than time averages can be used. Can use nonexchangeable functions, too.

5. Empirical Example

- Papke and Wooldridge (2008), Michigan School Reform.
- *math*4, a pass rate, is a fractional response.
- “Foundation Allowance” as an IV for average district spending.
- Other controls: enrollment, poverty rate, year effects.
- Uses a kinked relationship. IV is strong.
- $N = 501$, $T = 7$

Model:	**Linear**	**Linear**	**FProbit**		**FProbit**	
Estimation:	**FE**	**FEIV**	**PQMLE**		**PQMLE**	
	Coef	Coef	Coef	APE	Coef	APE
l*avgrexp*	.377	.420	.821	.277	.797	.269
	(.071)	(.115)	(.334)	(.112)	(.338)	(.114)
$\hat{v}_2$	—	$-.060$	$.076$		$-.666$	
	—	(.146)	(.145)		(.396)	
$\overline{lavgrexp}$?	—	—	Yes		No	

6. Extensions and Future Directions

- Extension to discrete EEVs. Lose identification without strong assumptions.

  - Can combine CRE and use one-step pooled quasi-MLE ("bivariate probit" is an example, as in Wooldridge (2014, J of E).

  - Use generalized residuals as the CFs, such as

$$\widehat{gr}_{it2} = y_{it2}\lambda(\mathbf{w}_{it}\hat{\boldsymbol{\theta}}_2) - (1 - y_{it2})\lambda(-\mathbf{w}_{it}\hat{\boldsymbol{\theta}}_2)$$

when y_{it2} follows a reduced form probit.

- Unbalanced panels. Condition on time averages and number of time periods and use complete cases (in reduced forms and CREs).
- Dynamic models with heterogeneity and EEVs (Giles and Murtazashvili, 2013, JEM).
- Resiliency to model misspecification? (CRE functions, control functions, and semiparametrics.)