Panel GLS unit root tests and common factors*

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Abstract

In this paper we propose GLS-based panel data unit root tests statistics that allow for cross-section dependence and multiple structural breaks in both the level and the slope of the time trend. The analysis covers both the case of (mild) heterogeneous and homogeneous break points. We evaluate the finite-sample properties of these statistics via a Monte Carlo simulation, considering both the known and unknown structural breaks cases. The paper illustrates the application of the proposed statistics analyzing the a panel of annual data of GDP for 19 developed OECD countries covering the period 1870-2008.

JEL Classification: C12, C22

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1 Introduction

Ever since Perron (1989) published his seminal paper on unit root tests and structural breaks, the interest on this field rose considerably. Failure to account for structural breaks leads to size distortions and potentially biased estimates of the parameters in which unit root tests build upon. This line of research began when the author showed that the Dickey-Fuller (DF) statistic is biased towards the null hypothesis of unit root if there is an external shock affecting the slope of the time series. He proposed different tests, consistent under both the null and the alternative hypotheses, with the condition that the structural break date is known *a priori*. This condition was later criticized by Christiano (1992) who argued that the date of the structural break is chosen based on pre-test examination of the data so that the analysis becomes conditional on the decision of the practitioner. As a result, the following studies took into consideration that the break date was determined endogenously and examples of such studies for one unknown structural break include Zivot and Andrews (1992), Perron and Vogelsang (1992), Banerjee, Lumsdaine and Stock (1992) and Perron (1997).

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The next line of research for univariate case includes studies for two or more structural breaks. Lumsdaine and Papell (1997) argued that unit root test results are sensitive to the number of assumed structural breaks and they extended the Zivot and Andrews (1992) analysis in order to account for two endogenously determined structural breaks. Note that the framework of Zivot and Andrews (1992) allows the structural break only under the alternative hypothesis. For the case when a series has a structural break under the null hypothesis, the rejection of the null might indicate that the time series is I(0) stationary with structural breaks when in fact it is I(1) non-stationary with structural breaks. Lee and Strazicich (2003) went one step further and proposed a minimum Lagrange Multiplier (LM) unit root test that allows for two unknown structural breaks under both the null and the alternative hypotheses. The authors compared their results with those of Lumsdaine and Papell (1997) using the same dataset and found that the statistic proposed by Lumsdaine and Papell (1997) tends to reject the null hypothesis of unit root more than the LM test.

Other researchers extended the previous analyses to more than two endogenously determined structural breaks. Ohara (1999) and Kapetanios (2005) generalized the Zivot and Andrews (1992) methodology to allow for m structural breaks but only under the alternative hypothesis of I(0) – see Perron (2006) for a complete overview. Carrion-i-Silvestre, Kim and Perron (2009, CKP hereafter) extended the unit root tests based on the GLS detrending procedure proposed in Ng and Perron (2001). Their framework allows for multiple structural breaks under both the null and alternative hypotheses. Also, they allow for structural breaks in both the level and the slope of the time trend of the series. It has been shown that these statistics have better size and power properties than those that allow for structural breaks only under the stationary alternative. In a recent paper, Westerlund (2012) extended the Amsler and Lee (1995) unit root test to allow for multiple structural breaks in the level of the data. Like in the CKP paper, the breakpoints are allowed under both the null and alternative hypotheses.

With the increasing development of panel data methods, the researchers extended the univariate analysis to panel data framework. However, these types of extensions are relatively limited. The first generation of panel data unit root and stationarity tests with structural breaks assumed that the units of the panel are cross-section independent. For example, Carrion-i-Silvestre, Del Barrio, and López-Bazo (2001) extended the panel data Dickey-Fuller (DF) unit root test in Harris and Tzavalis (1999) considering one structural break in the level of the time series. Another example of panel unit root test is the work of Im, Lee and Tieslau (2005) who extended the LM-based test while allowing for up to two level shifts. Finally, Carrion-i-Silvestre, Del Barrio, and López-Bazo (2005) developed a panel data stationarity test allowing for multiple structural breaks in both the intercept and/or the slope of the time series.

Note that these studies did not account for the cross-sectional dependence that plagued the earlier panel data studies, the so-called first-generation panel tests. In the recent years it was shown that the cross-section independence assumption is not realistic especially in country or regional studies. For example, one important problem that we have to deal nowadays is the increase in oil prices. As a result, many macroeconomic variables from one country are very close related with those from a neighboring country. That is, due to a common shock, the cross-section variables of the panel of countries are dependent on one another. If the cross-section dependence is not accounted for, it can cause biased and inconsistent estimates. So the next necessary step is taking into account the dependency between cross-sections while still allowing for structural breaks.

One example of such studies is that of Bai and Carrion-i-Silvestre (2009). The authors treated the cross-section dependency by using common factors originally proposed by Bai and Ng (2004). They proposed as a panel unit root test the square of the modified MSB test defined by Stock (1999) while allowing for multiple structural breaks. The test is invariant in the limit only to level shifts but not to structural breaks affecting the slope of the time trend. Therefore, the authors also proposed a simplified MSB test statistic that is invariant to both level and slope shifts, although the limiting distribution still depends on the number of structural breaks. Another example of a related work is that of Tam (2006) who proposed panel unit root tests that are an extension of the LM-based test and the combination tests of Maddala and Wu (1999) and Choi (2001). The author handles the impact of cross-section dependence by means of bootstrapping. Another study that extends a LM-based unit root test to panel data is the one by Westerlund (2012), who allows for multiple structural breaks in the level of the data. In order to estimate the number of the structural breaks and their location, Westerlund (2012) suggests a procedure based on outlier detection that is valid under both the null and alternative hypotheses, does not require *a priori* knowledge about the number or the location of the structural breaks, and is robust to cross-sectional dependence captured by common factors. However, the procedure is restricted to stationary errors with weak dependence at extreme levels. Im, Lee and Tieslau (2012) proposed another LM-based panel unit root test that allows for heterogeneous structural breaks in both the level and the slope of the time trend of the series. Their statistic depends only on the number of structural breaks but not on their size or location, and is invariant to nuisance parameters. The authors apply the cross-sectionally augmented ADF (CADF) regression of Pesaran (2007) to their tests as one possible means of correcting for cross-section dependence. Finally, Lee and Wu (2012) suggested a panel unit root test based on the generalized CADF procedure proposed by Pesaran (2007). They incorporate a single-frequency-component Fourier function that is used to approximate the unknown multiple structural breaks. The cross-sectional dependence is modeled by an unobservable I(0) stationary common factor. To the best of our knowledge, none of the existing panel studies that use GLS detrending in their estimation allows for structural breaks under both the null and the alternative hypotheses and in both the intercept and slope of the series.

In this paper, we propose several panel data unit root tests that are based on the GLS detrending procedure. The statistics are the extension of univariate CKP statistics to panel data. The new tests allow for multiple structural breaks that affect either the level and/or the slope of the time trend. Like in the CKP study, we allow structural breaks under both the null and the alternative hypotheses. Moreover, we deal with the cross-section dependence through the use of common factors. We then evaluate the finite-sample properties of our statistics via Monte Carlo simulations. Our simulation study shows that the tests perform well for the both cases of known and unknown structural breaks. Finally, we apply the proposed tests to a panel of annual per capita real GDP over the period 1870-2008 for 19 OECD countries.

The structure of this paper is as follows. In Section 2 we describe the model, while Section 3 presents the unit root test statistics that are investigated. The proposed panel statistics are shown in Section 5. Section 6 summarizes the Monte Carlo simulations results and the empirical application is carried out in Section 7. Finally, the paper concludes in Section 8.

2 The model

Let us consider the data generating process (DGP) given by the following system of equations:

$$
\begin{aligned}
y_{i,t} &= d_{i,t} + F_t'\delta_i + e_{i,t} && (1)\\
(I-L)\,F_t &= C(L)\,w_t && (2)\\
(1-\theta_i L)\,e_{i,t} &= B_i(L)\,\varepsilon_{i,t}, && (3)
\end{aligned}
$$

$i = 1, \ldots, N$, $t = 1, \ldots, T$, where the stochastic process $y_{i,t}$ is decomposed as the sum of a deterministic term $d_{i,t}$, a common factor component $F_t'\delta_i$ and the idiosyncratic stochastic component $e_{i,t}$. In this framework the cross-section dependence among time series in the panel data is driven by an approximate common factor model as in Bai and Ng (2002, 2004). F_t denotes a $(r \times 1)$ vector of unobserved common factors and δ_i is a $(r \times 1)$ vector of factor loadings. Note that F_t can be I(0), I(1) or a combination of I(0) and I(1) common factors depending on the rank of $C(1)$. For example, if $C(1) = 0$ then F_t is I(0). If the rank of $C(1)$ is r_1 then they are r_1 common stochastic trends and $r - r_1$ I(0) common factors. If $C(1)$ has full rank then F_t is $I(1)$.

Let $M < \infty$ be a generic positive number, independent of T and N and let $\|A\| = trace\,(A'A)^{1/2}$. We follow Bai and Ng (2004) and define the following assumptions:

Assumption A: (i) for non-random δ_i, $\|\delta_i\| \leq M$; for random δ_i, $E\|\delta_i\|^4 \leq M$. (ii) $\frac{1}{N}\sum_{i=1}^{N} \delta_i\delta_i' \xrightarrow{p} \Sigma_\delta$, a $(r \times r)$ positive definite matrix.

Assumption B: (i) $w_t \sim iid\,(0, \Sigma_w)$, $E\|w_t\|^4 \leq M$. (ii) $Var\,(\Delta F_t) = \sum_{j=0}^{\infty} C_j\Sigma_w C_j' > 0$. (iii) $\sum_{j=0}^{\infty} j\,\|C_j\| < M$. (iv) $C(1)$ has rank r_1, $0 \leq r_1 \leq r$.

Assumption C: (i) for each i, $\varepsilon_{i,t} \sim iid\,(0, \Sigma_{\varepsilon_i})$, $E\,|\varepsilon_{i,t}|^8 \leq M$. (ii) $Var\,(\Delta\varepsilon_{i,t}) = \sum_{j=0}^{\infty} B_{i,j}\Sigma_{\varepsilon_i}B_{i,j}' > 0$. (iii) $\sum_{j=0}^{\infty} j\,\|B_{i,j}\| < M$.

Assumption D: $\varepsilon_{i,t}$, w_t and δ_i are mutually independent.

Assumption E: $E\,\|F_0\| \leq M$, and for every $i = 1, \ldots, N$, $E\,|e_{i,0}| \leq M$.

Assumptions A and B implies the existence of r common factors. Assumption B permits a combination of I(0) and I(1) common factors in the model. Assumption C(i) allows some weak correlation in $(1-\theta_i L)\,e_{i,t}$, while C(ii) and C(iii) allow weak cross-section correlation. Assumption D states that the errors $\varepsilon_{i,t}$, w_t and δ_i are mutually independent across i and t. Assumption E defines the initial conditions.

The definition of the deterministic component in (1) gives rise to three different models. In Model 0, the multiple structural breaks occur in the intercept and it is known as the "level shift" model. Model I is known as the "slope change" model and allows for structural breaks only in the slope of the time trend. Finally, Model II allows for multiple structural breaks in both the intercept and the slope of the time trend. These models can be parameterized as

$$
d_{i,t} = \sum_{j=0}^{m} z_{i,t}'\left(T_{i,j}^0\right)\psi_{i,j} \equiv z_{i,t}'\left(\lambda_i^0\right)\psi_i,
$$

where $z_{i,t}\left(\lambda_i^0\right) = \left[z_{i,t}'\left(T_{i,0}^0\right), \ldots, z_{i,t}'\left(T_{i,m}^0\right)\right]'$, $\psi_i = \left(\psi_{i,0}', \ldots, \psi_{i,m}'\right)'$ and

$$z_{i,t}\left(T_{i,j}^0\right) = \begin{cases} DU_{i,t}\left(T_{i,j}^0\right) & \text{for Model 0} \\ DT_{i,t}\left(T_{i,j}^0\right) & \text{for Model I} \\ \left(DU_{i,t}\left(T_{i,j}^0\right), DT_{i,t}\left(T_{i,j}^0\right)\right)' & \text{for Model II} \end{cases},$$

and

$$\psi_{i,j} = \begin{cases} \mu_{i,j} & \text{for Model 0} \\ \beta_{i,j} & \text{for Model I} \\ \left(\mu_{i,j}, \beta_{i,j}\right)' & \text{for Model II} \end{cases},$$

$0 \leq j \leq m$, with $DU_{i,t}\left(T_{i,j}^0\right) = 1$ and $DT_{i,t}\left(T_{i,j}^0\right) = t - T_{i,j}^0$ for $t > T_{i,j}^0$ and 0 otherwise, where $T_{i,j}^0 = \left\lfloor T\lambda_{i,j}^0\right\rfloor$ represents the true break date for the i-th individual – $\lfloor\cdot\rfloor$ denotes the integer part of the element between brackets – λ_i^0 is a $(m \times 1)$-vector with the true break fractions and with the convention that $T_{i,0}^0 = 0\ \forall i$. It is worth noticing that the use of "0" as a superscript indicates that the structural breaks are known a priori – the case of unknown structural breaks is addressed below.[1]

As for the break dates, the model that we specify assumes that the break dates admit certain degree of heterogeneity through the definition of

$$T_{i,j}^0 = T_j^0 + v_{i,j}, \tag{4}$$

with $v_{i,j} \sim iid\left(0, \sigma_{i,j}^2\right)\ \forall i, j$, i.e., the break dates are assumed to depart from a common break dates up to a bounded quantity. Note that in the limit, the fraction parameters are common to all individuals since

$$\begin{aligned} \lambda_{i,j}^0 &= T_{i,j}^0/T \\ &= T_j^0/T + O_p\left(T^{-1}\right) \xrightarrow{p} \lambda_j^0, \end{aligned}$$

where $\xrightarrow{p}$ denotes convergence in probability. Consequently, although in finite samples the break dates are allowed to be mildly heterogeneous across individuals, in the limit the break fraction vector is common to all individuals, i.e., $\lambda_i^0 \xrightarrow{p} \lambda^0 = (\lambda_1^0, \ldots, \lambda_m^0)'$. It is worth mentioning that it is also possible to impose here the constraint that the break points are common to all individuals if we set $v_{i,j} = 0\ \forall i, j$ in (4), so that $\lambda_{i,j}^0 = \lambda_j^0\ \forall i, j$.

The GLS-detrended unit root statistics use the transformed data $y_{i,t}^{\bar{\alpha}}$ and $z_{i,t}^{\bar{\alpha}}\left(\lambda_i^0\right)$, which is defined as $y_{i,1}^{\bar{\alpha}} = y_{i,1}$ and $z_{i,1}^{\bar{\alpha}}\left(\lambda_i^0\right) = z_{i,1}\left(\lambda_i^0\right)$ for $t = 1$, and $y_{i,t}^{\bar{\alpha}} = (1 - \bar{\alpha}L)\, y_{i,t}$ and $z_{i,t}^{\bar{\alpha}}\left(\lambda_i^0\right) = (1 - \bar{\alpha}L)\, z_{i,t}\left(\lambda_i^0\right)$ for $t = 2, \ldots, T$, $i = 1, \ldots, N$, with $\bar{\alpha} = 1 + \bar{c}\left(\lambda^0\right)/T$ and $\bar{c}\left(\lambda^0\right)$ being the non-centrality parameter defined in CKP. Let $\tilde{\psi}_i$ be the estimator that minimizes the sum of squared residuals

$$S(\psi_i, \lambda_i^0) = \left(y_{i,t}^{\bar{\alpha}} - z_{i,t}^{\bar{\alpha}\prime}\left(\lambda_i^0\right)\psi_i\right)'\left(y_{i,t}^{\bar{\alpha}_i} - z_{i,t}^{\bar{\alpha}\prime}\left(\lambda_i^0\right)\psi_i\right).$$

Using these estimated parameters we can construct the GLS-detrended variable $\tilde{y}_{i,t} = y_{i,t} - z_{i,t}'\left(\lambda_i^0\right)\tilde{\psi}_i$ and compute its first difference

$$\begin{aligned} \Delta\tilde{y}_i &= \Delta y_i - \Delta z_i'\left(\lambda_i^0\right)\hat{\psi}_i \\ &= -\Delta z_i\left(\lambda_i^0\right)\left(\hat{\psi}_i - \psi_i\right) + \Delta F\delta_i + \Delta e_i \\ &= -\Delta z_i\left(\lambda_i^0\right)\left(z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right)\right)^{-1} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right)\left(F^{\bar{\alpha}}\delta_i + e_i^{\bar{\alpha}}\right) + \Delta F\delta_i + \Delta e_i \\ &= f\delta_i + \xi_i, \end{aligned} \tag{5}$$

[1] Although we deal with three different specifications involving structural breaks, our setup can also be particularized to the case of no structural breaks considering $m_i = 0$.

where $\xi_i = \Delta e_i - \Delta z_i \left(\lambda_i^0\right) (z_i^{\bar{\alpha}\prime} \left(\lambda_i^0\right) z_i^{\bar{\alpha}} \left(\lambda_i^0\right))^{-1} z_i^{\bar{\alpha}\prime} \left(\lambda_i^0\right) e_i^{\bar{\alpha}}$ and $f = \Delta F - \Delta z_i \left(\lambda_i^0\right) (z_i^{\bar{\alpha}\prime} \left(\lambda_i^0\right) z_i^{\bar{\alpha}} \left(\lambda_i^0\right))^{-1} z_i^{\bar{\alpha}\prime} \left(\lambda_i^0\right) F^{\bar{\alpha}_i}$. In this case (5) is a factor model with the new observable variables $\Delta\tilde{y}_{i,t}$ and the estimation of the common factors and factor loadings can be done as in Bai and Ng (2004) using principal components. Let

$$\Delta\tilde{y} = (\Delta\tilde{y}_1, \Delta\tilde{y}_2, ..., \Delta\tilde{y}_N),$$

be the $(T-1) \times N$ data matrix. The estimated principal components of $\Delta F = (\Delta F_2, \Delta F_3, \ldots, \Delta F_T)$, denoted as $\Delta\tilde{F}$, are $\sqrt{T-1}$ times the r eigenvectors corresponding to the first r largest eigenvalues of the $(T-1) \times (T-1)$ matrix $\Delta\tilde{y}\Delta\tilde{y}'$, under the normalization $\Delta\tilde{F}\Delta\tilde{F}'/(T-1) = I_r$. The estimated loading matrix is $\tilde{\Lambda} = \Delta\tilde{y}'\Delta\tilde{F}/(T-1)$, from which we can define the estimated residuals as

$$\Delta\tilde{e}_{i,t} = \Delta\tilde{y}_{i,t} - \Delta\tilde{F}_t'\tilde{\delta}_i. \tag{6}$$

We can recover the idiosyncratic disturbance terms through cumulation, i.e.,

$$\tilde{e}_{i,t} = \sum_{s=2}^{t} \Delta\tilde{e}_{i,s},$$

whereas the common factors are estimated in the same fashion:

$$\tilde{F}_t = \sum_{s=2}^{t} \Delta\tilde{f}_s.$$

Using these two estimated components we can assess the source of potential I(1) non-stationarity of the observable variables.

3 Unit root test statistics

The order of integration of each component can be analyzed using the modified test statistics in Ng and Perron (2001) – hereafter, M-type test statistics. Thus, if we focus on the idiosyncratic component, the M-type unit root tests statistics are defined as:

$$MSB_i^{GLS} = \left(s_i^{-2}T^{-2}\sum_{t=1}^{T}\tilde{e}_{i,t-1}^2\right)^{1/2} \tag{7}$$

$$MZ_{i,\alpha}^{GLS} = \left(T^{-1}\tilde{e}_{i,T}^2 - s_i^2\right)\left(2T^{-2}\sum_{t=1}^{T}\tilde{e}_{i,t-1}^2\right)^{-1} \tag{8}$$

$$MZ_{i,t}^{GLS} = \left(T^{-1}\tilde{e}_{i,T}^2 - s_i^2\right)\left(4s_i^2T^{-2}\sum_{t=1}^{T}\tilde{e}_{i,t-1}^2\right)^{-1/2} \tag{9}$$

where $s_i^2 = (T-k_i)^{-1}\left(\sum_{t=k_i+1}^{T}\tilde{u}_{t,k_i}^2\right)/\left(1-\sum_{j=1}^{k_i}\tilde{b}_{i,j}\right)^2$ and k_i is the order of the autoregression selected using the information criteria proposed by Ng and Perron (2001) and Perron and Qu (2007). The terms $\tilde{u}_{t,k_i}$ and $\tilde{b}_{i,j}$ are the OLS estimated coefficients from the regression

$$\Delta\tilde{y}_{i,t} = b_{i,0}\tilde{y}_{i,t-1} + \sum_{j=1}^{k_i} b_{i,j}\Delta\tilde{y}_{i,t-j} + u_{t,k_i}.$$

Using these statistics we can test the hypotheses that $e_{i,t}$ is I(1) against the alternative hypothesis that $e_{i,t}$ is I(0), i.e.,

$$\left\{ \begin{array}{ll} H_0: & \theta_i = 1 \\ H_1: & \theta_i < 1 \end{array} \right. .$$

As for the common factors, in the case where there is only one ($r = 1$) common factor, we can proceed to test its order of integration using the M-type test statistics defined above, once it has been GLS-detrended. The GLS detrending is performed using the vector of regressors $z_t\left(\lambda^0\right)$ that define the break dates $T_j^0 = E\left(T_{i,j}^0\right) = N^{-1}\sum_{i=1}^{N} T_{i,j}^0$, $j = 0, 1, \ldots, m$, which makes use of the mild heterogeneous property for the break dates. The GLS-detrended estimated common factor is defined as $\tilde{F}_t^d = \tilde{F}_t - z_t'\left(\lambda^0\right)\tilde{\psi}$, where $\tilde{\psi}$ is the quasi-GLS estimation of the parameters. Then, the unit root tests given in (7)-(9) can be computed using $\tilde{F}_t^d$ instead of $\tilde{e}_{i,t}$ – the corresponding test statistics are denoted as MSB_F^{GLS}, $MZ_{F,\alpha}^{GLS}$ and $MZ_{F,t}^{GLS}$.

When there is more than one common factor ($r > 1$), we can assess how many common factors are I(1) and I(0) using the MQ test statistics in Bai and Ng (2004). Start with $q = r$ and proceed in three stages:

1. Let $\tilde{\beta}_\perp$ be the q eigenvectors associated with the q largest eigenvalues of $T^{-2}\sum_{t=2}^{T}\tilde{F}_t^d\tilde{F}_t^{d\prime}$.
2. Let $\tilde{Y}_t^d = \tilde{\beta}_\perp\tilde{F}_t^d$, from which we can define two statistics:
   (a) Let $K(j) = 1 - j/(J+1)$, $j = 0, 1, 2, \ldots, J$:
      i. Let $\tilde{\xi}_t^d$ be the residuals from estimating a first-order VAR in $\tilde{Y}_t^d$, and let
      $$\tilde{\Sigma}_1^d = \sum_{j=1}^{J} K(j)\left(T^{-1}\sum_{t=2}^{T}\tilde{\xi}_{t-j}^d\tilde{\xi}_t^{d\prime}\right).$$
      ii. Let $\tilde{v}_c^d(q) = \frac{1}{2}\left[\sum_{t=2}^{T}\left(\tilde{Y}_t^d\tilde{Y}_{t-1}^{d\prime} + \tilde{Y}_{t-1}^d\tilde{Y}_t^{d\prime}\right) - T\left(\tilde{\Sigma}_1^d + \tilde{\Sigma}_1^{d\prime}\right)\right]\left(\sum_{t=2}^{T}\tilde{Y}_{t-1}^d\tilde{Y}_{t-1}^{d\prime}\right)^{-1}$.
      iii. Define $MQ_c^d(q) = T\left[\tilde{v}_c^d(q) - 1\right]$ for the case of no change in the trend and $MQ_c^d\left(q, \lambda^0\right) = T\left[\tilde{v}_c^d\left(q, \lambda^0\right) - 1\right]$ for the case of changes in the trend.
   (b) For p fixed that does not depend on N and T:
      i. Estimate a VAR of order p in $\Delta\tilde{Y}_t^d$ to obtain $\tilde{\Pi}(L) = I_q - \tilde{\Pi}_1 L - \ldots - \tilde{\Pi}_p L^p$. Filter $\tilde{Y}_t^d$ by $\tilde{\Pi}(L)$ to get $\tilde{y}_t^d = \tilde{\Pi}(L)\tilde{Y}_t^d$.
      ii. Let $\tilde{v}_f^d(q)$ be the smallest eigenvalue of
      $$\Phi_f^d = \frac{1}{2}\left[\sum_{t=2}^{T}\left(\tilde{y}_t^d\tilde{y}_{t-1}^{d\prime} + \tilde{y}_{t-1}^d\tilde{y}_t^{d\prime}\right)\right]\left(\sum_{t=2}^{T}\tilde{y}_{t-1}^d\tilde{y}_{t-1}^{d\prime}\right)^{-1}.$$
      iii. Define the statistic $MQ_f^d(q) = T\left[\tilde{v}_f^d(q) - 1\right]$ for the case of no change in the trend and $MQ_f^d\left(q, \lambda^0\right) = T\left[\tilde{v}_f^d\left(q, \lambda^0\right) - 1\right]$ for the case of changes in the trend.
3. If $H_0 : r_1 = q$ is rejected, set $q = q - 1$ and return to the first step. Otherwise, $\tilde{r}_1 = q$ and stop.

The limiting distribution of these unit root test statistics is presented in the following theorem.

Theorem 1 *Let $y_{i,t}$, $i = 1, \ldots, N$, $t = 1, \ldots, T$, be a stochastic process with the DGP given by (1) to (3) and satisfying Assumptions A to E. Also, define $\bar{\alpha} = 1 + \bar{c}\left(\lambda^0\right)/T$, let s_i^2 be a consistent estimate of σ_i^2 and let k_i be chosen in a way that $k_i \to \infty$ and $k_i^3/\min[N, T] \to 0$. Then as $N, T \to \infty$ with $N/T \to 0$*
(a) the statistics applied to the idiosyncratic component converge to:

Model 0:

$$
\begin{aligned}
MSB_i^{GLS} &\Rightarrow \left(\int_0^1 V_{i,c,\bar{c}}(s)^2\, ds\right)^{1/2} \\
MZ_{\alpha,i}^{GLS} &\Rightarrow 0.5\left(V_{i,c,\bar{c}}(1)^2 - 1\right)\left(\int_0^1 V_{i,c,\bar{c}}(s)^2\, ds\right)^{-1} \\
MZ_{t,i}^{GLS} &\Rightarrow 0.5\left(V_{i,c,\bar{c}}(1)^2 - 1\right)\left(\int_0^1 V_{i,c,\bar{c}}(s)^2\, ds\right)^{-1/2}
\end{aligned}
$$

Models I and II:

$$
\begin{aligned}
MSB_i^{GLS} &\Rightarrow \left(\int_0^1 V_{i,c,\bar{c}}\left(s, \lambda^0\right)^2 ds\right)^{1/2} \\
MZ_{\alpha,i}^{GLS} &\Rightarrow 0.5\left(V_{i,c,\bar{c}}\left(1, \lambda^0\right)^2 - 1\right)\left(\int_0^1 V_{i,c,\bar{c}}\left(s, \lambda^0\right)^2 ds\right)^{-1} \\
MZ_{t,i}^{GLS} &\Rightarrow 0.5\left(V_{i,c,\bar{c}}\left(1, \lambda^0\right)^2 - 1\right)\left(\int_0^1 V_{i,c,\bar{c}}\left(s, \lambda^0\right)^2 ds\right)^{-1/2},
\end{aligned}
$$

where $\Rightarrow$ denotes weak convergence to the associated measure of probability, $V_{i,c,\bar{c}}(s) = W_{i,c}(s) - s\left(bW_{i,c}(1) + 3(1-b)\int_0^1 uW_{i,c}(u)\, du\right)$, $b = (1-\bar{c})/(1-\bar{c}+\bar{c}^2/3)$, $V_{i,c,\bar{c}}\left(s, \lambda^0\right) = W_{i,c}(s) - z_2(s)\, A\left(\lambda^0\right)^{-1} \bar{V}_i\left(\lambda^0\right)$, $W_{i,c}(s)$ is an Ornstein-Uhlenbeck process and the terms $A\left(\lambda^0\right)$ and $\bar{V}_i\left(\lambda^0\right)$ for each cross-section are defined in the Appendix.
(b) When $r = 1$, the limiting distribution for the MSB_F^{GLS}, $MZ_{F,\alpha}^{GLS}$ and $MZ_{F,t}^{GLS}$ test statistics for the different model specifications is the same as the one given by MSB_i^{GLS}, $MZ_{i,\alpha}^{GLS}$ and $MZ_{i,t}^{GLS}$, respectively.
(c) When $r > 1$, let $V_{q,c,\bar{c}}(s)$ and $V_{q,c,\bar{c}}\left(s, \lambda^0\right)$ be q-vectors with elements defined by $V_{j,c,\bar{c}}(s) = W_{j,c}(s) - s\left(bW_{j,c}(1) + 3(1-b)\int_0^1 uW_{j,c}(u)\, du\right)$ and $V_{j,c,\bar{c}}\left(s, \lambda^0\right) = W_{j,c}(s) - z_2(s)\, A\left(\lambda^0\right)^{-1} \bar{V}_j\left(\lambda^0\right)$, $j = 1, \ldots, q$, respectively.
For Model 0, let $v_*^d(q)$ be the smallest eigenvalues of

$$
\Phi_*^d = \frac{1}{2}\left[V_{q,c,\bar{c}}(1)\, V_{q,c,\bar{c}}(1)' - I_p\right]\left[\int_0^1 V_{q,c,\bar{c}}(s)\, V_{q,c,\bar{c}}(s)'\, ds\right]^{-1}.
$$

For Models I and II, let $v_*^d\left(q, \lambda^0\right)$ be the smallest eigenvalues of

$$
\Phi_*^d(\lambda) = \frac{1}{2}\left[V_{q,c,\bar{c}}\left(1, \lambda^0\right) V_{q,c,\bar{c}}\left(1, \lambda^0\right)' - I_p\right]\left[\int_0^1 V_{q,c,\bar{c}}\left(s, \lambda^0\right) V_{q,c,\bar{c}}\left(s, \lambda^0\right)' ds\right]^{-1}.
$$

(c.1) Let J be the truncation lag of the Bartlett kernel, chosen such that $J \to \infty$ and $J/\min\left[\sqrt{N}, \sqrt{T}\right] \to 0$. Then, under the null hypothesis that F_t has q stochastic trends, $MQ_c^d(q) \Rightarrow v_*^d(q)$ and $MQ_c^d\left(q, \lambda^0\right) \Rightarrow v_*^d\left(q, \lambda^0\right)$.
(c.2) Under the null hypothesis that F_t has q stochastic trends with a finite VAR($\bar{p}$) representation and a VAR(p) is estimated with $p \geq \bar{p}$, $MQ_f^d(q) \Rightarrow v_*^d(q)$ and $MQ_f^d\left(q, \lambda^0\right) \Rightarrow v_*^d\left(q, \lambda^0\right)$.

The asymptotic critical values for the MSB_i^{GLS}, $MZ_{i,\alpha}^{GLS}$ and $MZ_{i,t}^{GLS}$ – and the ones for the common factor, MSB_F^{GLS}, $MZ_{F,\alpha}^{GLS}$ and $MZ_{F,t}^{GLS}$ – can be found in Carrion-i-Silvestre et al. (2009). The asymptotic critical values for the MQ test statistics can be found in Table 1 for Model 0 and in Table 2 for Models I and II. For further developments, we also have computed the asymptotic mean and variance of the limiting distribution of the different test statistics reported in Theorem 1.

Finally, the model that we have specified assumes that the number of common factors is known. In practice we will need to estimate it using, for instance, the information criteria proposed in Bai and Ng (2002, 2004). We will analyze the performance of the use of these information criteria in the Monte Carlo simulation section.

4 Unknown structural breaks

So far, we have assumed that the vector of break points is known. This assumption might be feasible in some cases, where panels of variables such as, for instance, real exchange rates panels are analyzed and it is known that there is an important event that have affected the time series – in this case, the euro currency birth. However, there might be some cases where this assumption cannot be made and the date of the structural breaks needs to be estimated. In this section we present two strategies that can be followed in order to select the date of the structural breaks. Throughout this section, we assume that the number of structural breaks (m) is known and common to all individuals. In principle, it would be possible to specify a maximum number of structural breaks ($m_{\max}$) and estimate the number of structural breaks ($\tilde{m}$) using a panel Bayesian information criterion such as the one proposed in Bai and Ng (2002, 2004).

Individual estimation. In this case, we proceed to estimate the date of the break points for each time series, without taking into account the other time series of the panel data set. The estimation of the unknown structural breaks for each time series relies on the procedure proposed in Carrion-i-Silvestre et al. (2009) and consists of the following steps:

1. For a given value of m, compute an initial educated estimation of the date of the break points $\tilde{T}_i = \left(\tilde{T}_{i,1}, \ldots, \tilde{T}_{i,m}\right)'$ and the vector of parameters $\tilde{\psi}_i = \left(\tilde{\psi}'_{i,0}, \ldots, \tilde{\psi}'_{i,m}\right)'$ specifying the model
$$y_{i,t} = z'_{i,t}\left(\lambda_i^0\right)\psi_i + u_{i,t},$$
where the OLS estimation procedure is used to obtain the estimates. The estimates that are obtained in this step are the ones drawn from the minimization of the sum of squared residuals (SSR)

2. Obtain $\bar{c}(\tilde{\lambda}_i)$ using the estimates obtained in the previous step

3. Compute the quasi-difference of the variables and proceed to compute the GLS estimates of the parameters and break dates minimizing the restricted sum of squared residuals (RSSR) – see Carrion-i-Silvestre et al. (2009) for further details

4. Repeat steps 2 and 3 until convergence is achieved and store the estimated break dates

5. Compute the GLS detrended variable $\tilde{y}_{i,t}$ using the final estimates of the break points and the parameters of the model

6. Estimate the common factors, the factor loadings and the idiosyncratic disturbance terms using the method of principal components described in Section 2

7. Test the order of integration of the different components using the unit root test statistics proposed in Section 3

Homogeneous estimation. In this case we proceed to estimate the break points imposing the constraint that they are common (equal) to all time series in the panel data set, without allowing for the mild heterogeneity that was specified in the model – i.e., setting $v_{i,j} = 0\ \forall i, j$ in (4). The implementation of this estimation procedure is similar to the previous one, but where the break points are obtained from the global minimization of either the SSR – if we are in step 1 of the previous algorithm – or the RSSR – if we are in step 3 of the previous algorithm. To be specific, the steps are the following ones:

1. For a given value of m, we define all possible combinations of break dates. For a given combination of break dates T_k, we estimate the model

   $$y_{i,t} = z'_{i,t}\left(\lambda_k^0\right)\psi_i + u_{i,t},$$

   for each time series and compute the associated SSR – which is denoted by $SSR_i\left(T_k\right)$. The global SSR for a given combination of break dates is given by $GSSR\left(T_k\right) = \sum_{i=1}^{N} SSR_i\left(T_k\right)$, which delivers the estimated common break points as the argument that minimizes the GSSR, i.e., $\tilde{T} = \arg\min_{\lambda\in\Lambda(\varepsilon)} GSSR\left(T_k\right)$ where the infimum is taken over all possible break fractions defined on the set

   $$\Lambda\left(\varepsilon\right) = \left\{\left(\lambda_1, \ldots, \lambda_m\right); \left|\lambda_{j+1} - \lambda_j\right| \geq \varepsilon\ \left(j = 1, \ldots, m-1\right), \lambda_1 \geq \varepsilon, \lambda_m \geq 1-\varepsilon\right\},$$

   with ε some trimming that defines the minimal length of a segment – a usual value in the literature is $\varepsilon = 0.15$.

2. Obtain $\bar{c}(\tilde{\lambda})$ using the estimates obtained in the previous step

3. Compute the quasi-difference of the variables and proceed to compute the GLS estimates of the parameters and break points minimizing the restricted global sum of squared residuals (RGSSR), which is obtained as $GRSSR\left(T_k\right) = \sum_{i=1}^{N} RSSR_i\left(T_k\right)$, with the RSSR given in Carrion-i-Silvestre et al. (2009)

4. Repeat steps 2 and 3 until convergence is achieved and store the estimated common break points

5. Compute the GLS detrended variable $\tilde{y}_{i,t}$ using the final estimates of the common break points and the parameters of the model

6. Estimate the common factors, the factor loadings and the idiosyncratic disturbance terms using the method of principal components described in Section 2

7. Test the order of integration of the different components using the unit root test statistics proposed in Section 3

5 Panel data unit root test statistics

Although the analysis that has been conducted so far allow us to test the order of integration of the different stochastic processes involved in the model at a unit-by-unit level, it is possible, in principle, to improve the performance of the statistical inference combining the individual test statistic. In order to pool the individual test statistics we require to introduce the additional assumption of cross-section independence of the idiosyncratic disturbance terms $e_{i,t}$. This assumption makes the individual test statistics to be cross-sectionally independent.

We are interested in testing the null hypothesis that all units are I(1) non-stationary against the alternative hypothesis that at least one unit is I(0) stationary, i.e.,

$$\left\{ \begin{array}{ll} H_0: & \theta_i = 1 \; \forall i \\ H_1: & \theta_i < 1 \text{ for some } i \end{array} \right. . \tag{10}$$

We define M in a generic way to define one of the M-type unit root test statistic that we have considered, i.e., for a given unit i we can compute any of the statistics $M_i = \{MSB_i^{GLS}, MZ_{i,\alpha}^{GLS}, MZ_{i,t}^{GLS}\}$. The first way to define a pool panel data test statistic bases on the standardized mean of the individual statistics

$$M_m = \frac{\sqrt{N}(\overline{M_i} - \bar{\zeta}^M))}{\bar{\nu}^M} \rightarrow N(0,1),$$

where $\overline{M} = N^{-1}\sum_{i=1}^{N} M_i$, $\bar{\zeta}^M = N^{-1}\sum_{i=1}^{N} \zeta_i^M$ and $\bar{\nu}^{M^2} = N^{-1}\sum_{i=1}^{N} \nu_i^{M^2}$, where ζ_i^M and $\nu_i^{M^2}$ are the mean and the variance of the M_i statistic, $M = \{MSB^{GLS}, MZ_\alpha^{GLS}, MZ_t^{GLS}\}$.

The next three panel data tests are based on the combination of the individual p-values. Bai and Ng (2004) noted that pooling based on the p-values not only can be used on unbalanced panels but it has the advantage of allowing heterogeneity across units. Maddala and Wu (1999) define the panel data Fisher-type statistic that can be applied to small cross-sections

$$P^M = -2\sum_{i=1}^{N} \ln \tilde{\varphi}_i^M \sim \chi^2_{2N}, \tag{11}$$

where φ_i^M denotes the p-value of the M_i statistic, $M = \{MSB^{GLS}, MZ_\alpha^{GLS}, MZ_t^{GLS}\}$.

Choi (2001) goes one step further and proposes following test that is valid for $N \rightarrow \infty$:

$$P_m^M = -\frac{1}{\sqrt{N}}\sum_{i=1}^{N} \left(\ln \tilde{\varphi}_i^M + 1\right) \rightarrow N(0,1),$$

whereas Choi (2001) proposes the following pool test statistic:

$$Z^M = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} \Phi^{-1}\left(\tilde{\varphi}_i^M\right) \rightarrow N(0,1),$$

where $\Phi(\cdot)$ denotes the standard Normal cumulative distribution function, $M = \{MSB^{GLS}, MZ_\alpha^{GLS}, MZ_t^{GLS}\}$.

The simulated mean and variance of all the M-class tests that are needed in order to compute the panel unit root statistics are presented in Table 3. We also simulated the p-values for the M-class tests and they are available upon request.

6 Monte Carlo simulations

It is standard practice in econometrics to conduct a series of simulations in order to illustrate the finite sample properties of the proposed statistics. We first cover the cases with a single and then multiple structural breaks when the location of the potential structural break is known. Later, we relax the assumption of known structural breaks and we show the simulation results for both a single and multiple endogenous structural breaks. The nominal size of the statistics is set at the 5% level of significance. We present the results of the most general model specification (Model II) with structural breaks in both the slope and the trend. In addition, we focus on the first estimation procedure that has been described above, which bases the results on the individual estimation of the break points. All simulations are performed in GAUSS and the Monte Carlo results reported below are obtained using $1,000$ replications.

6.1 Known structural breaks

We begin the analysis by considering the performance of the panel data tests for the case of known single structural break. We account for cross-section dependence by using the common factor structure. The data generating process consists of the following system of equations:

$$
\begin{aligned}
y_{i,t} &= d_{i,t} + F_t'\delta_i + e_{i,t} && (12)\\
d_{i,t} &= \alpha_{i,1} DU_{i,t}\left(T_1^0\right) + \beta_{i,1} DT_{i,t}\left(T_1^0\right) && (13)\\
F_t &= \rho F_{t-1} + w_t && (14)\\
e_{i,t} &= \theta_i e_{i,t-1} + \varepsilon_{i,t} && (15)
\end{aligned}
$$

where $i = 1, \ldots, N$, $t = 1, \ldots, T$, $\varepsilon_{i,t} \sim iid\ N(0,1)$ and $w_t \sim iid\ N(0, \sigma_F^2)$. For the simulation of the common factor component we specify $\delta_i \sim N(1,1)$, $\rho = \{0.9, 0.95, 1\}$ and $\sigma_F^2 = \{0.5, 1, 10\}$. The number of common factors is estimated using the panel Bayesian information criterion (BIC) in Bai and Ng (2002). We set the autoregressive parameter $\theta_i = 1\ \forall i$. The data is generated with $N = 20$ cross-sectional units and all combinations of time series observations $T = \{50, 100, 200\}$. Due to space constraints we only present results for $\lambda^0 = 0.5$ with the note that we obtain similar results for $\lambda^0 = \{0.3, 0.7\}$. The full set of results are available upon request. Table 3 presents the mean and the variance of M-class statistics for all time dimensions T. The simulated size of the panel tests is summarized in Table 5. We can see that all tests perform well since the empirical size is really close to the nominal size of 5%. It is interesting to note that the results are similar for all panel statistics regardless of the order of integration of the common factors. Also, the size does not appear to be affected by the changes in T or σ_F^2. Based on these results we can infer that the performance of the panel tests for the case of one known structural break is good with almost no size distortions.

Next, we extend the analysis of panel data unit root tests for the case of two known structural breaks. The DGP has the following form:

$$
\begin{aligned}
y_{i,t} &= d_{i,t} + F_t'\delta_i + e_{i,t} && (16)\\
d_{i,t} &= \alpha_{i,1} DU_{i,t}\left(T_1^0\right) + \alpha_{i,2} DU_{i,t}\left(T_2^0\right) + \beta_{i,1} DT_{i,t}\left(T_1^0\right) + \beta_{i,2} DT_{i,t}\left(T_2^0\right) && (17)\\
F_t &= \rho F_{t-1} + w_t && (18)\\
e_{i,t} &= \theta_i e_{i,t-1} + \varepsilon_{i,t} && (19)
\end{aligned}
$$

where $i = 1, \ldots, N$, $t = 1, \ldots, T$, $\varepsilon_{i,t} \sim iid\ N(0,1)$ and $w_t \sim iid\ N(0, \sigma_F^2)$. As in the previous case, $\delta_i \sim N(1,1)$, $\rho = \{0.9, 0.95, 1\}$ and $\sigma_F^2 = \{0.5, 1, 10\}$. The sample size is $T = \{50, 100, 200\}$ and $N = 20$. Since we analyze the case with two structural breaks, we set $\lambda_1^0 = 0.3$ and $\lambda_2^0 = 0.7$.

The results on the empirical size of the panel unit root tests are reported in Table 6. We observe that the statistics maintain the nominal level well. The P_m panel statistic based on MSB^{GLS} and MZ_α^{GLS} tests tends to over-reject for $T = 100$ but its empirical size approaches the nominal size as T increases. Overall, for the case of two known structural breaks the statistics still perform well with very small size distortions.

6.2 Unknown structural breaks

In the previous sections, we assumed that the timing of the structural breaks is known. This is not always the case. Actually, many times, the researcher does not know *a priori* the location of the structural breaks and he/she needs the tools to do the proper analysis. Therefore, taking that into consideration, we simulate first the case when there is only one structural break and later on, we extend the analysis to the case with two structural breaks. The main difference between the known and unknown structural break cases consists in estimating the structural breaks. In the case of the unknown structural breaks, we estimate the location of the structural breaks through the global minimization of the RSSR of the GLS-detrended model presented in Section **??**. More explicitly, we estimate the location of unknown structural break by implementing the steps 1 to 8 shown on pages 7-8.

The data generating process for the case with one unknown structural break is the same like the one in Section 6.1 for one known structural break. More exactly, the DGP consists of equations (12) through (15). As in the case with known structural break, we present the results for $N = 20$ cross-sections and all combinations of time series dimensions $T = \{50, 100, 200\}$. We keep the same specification for the common factor component, therefore we set $\delta_i \sim N(1,1)$, $\rho = \{0.9, 0.95, 1\}$ and $\sigma_F^2 = \{0.5, 1, 10\}$ and we set the autoregressive parameter θ at 1. The results contained in Table 7 are summarized as follows. Overall, the performance of the statistics for the case of one unknown structural break is similar to that for the case of known structural break with a few exceptions. For $T = \{50, 100\}$ and $\sigma_F^2 = 10$, we can see that the empirical size is slightly above the 5% nominal size. This indicates a tendency of all the panel statistics to over-reject the null hypothesis of unit root. Also, for $T = 200$, the panel statistics P and P_m show small size distortion. More exactly, the values of these sizes indicate a under-rejection of the null hypothesis. However, the size of the panel statistics MSB_m and Z is really close to the nominal size. This indicates a good performance of these panel statistics. Overall, the simulations for the case of one unknown structural break lead us to conclude that the panel statistics perform rather well with small size distortions.

In the next step, we present the simulation results for two unknown structural breaks. The data generating process is the same as for the case with two known structural breaks and consists of Equations (16) through (19). Table 8 presents the empirical size of the panel statistics for the case of two unknown structural breaks. For the majority of cases, the size of the simulated statistics is close to the nominal size. There are a few exception when the size is slightly bigger or smaller than the 5% nominal size. For example, we can see a similar trend for all statistics when $T = 50$ and $\sigma_F^2 = 10$. More exactly, these values point out to a over-rejection of the null hypothesis of unit root. However, unlike the results for the case of one unknown structural break for $T = 100$ and $\sigma_F^2 = 10$, the tests have little size distortions for the case of two unknown structural breaks. The MSB_m panel statistic based on MSB^{GLS} test performs rather well for $T = 100$ but over-rejects for $T = 200$ and under-rejects for $T = 50$ when $\sigma_F^2 = \{0.5, 1\}$. The P and P_m statistics show moderate size distortions for $T = 200$. Their values of the size below the nominal size point to a under-rejection of the null hypothesis of unit root. The results for the size of Z test indicate that the test has a good performance overall with the exception when $T = 50$ and $\sigma_F^2 = 10$ mentioned earlier. Overall, we observe that in a few cases the statistics suffer from moderate size distortions but for the majority of cases the tests maintain the nominal level well.

7 Empirical application

In this section, we present an empirical application of the panel tests described in the previous sections. We use the Maddison dataset used by Dawson and Strazicich (2010) and Kejriwal and Lopez (2010). It consists of annual time series per capita real GDP over the period 1870-2008 for 19 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States. Note that the data are expressed in 1990 Geary-Ghamis dollars. The logarithm of per capita GDP is the output throughout the rest of the paper. For more details about the data, see Maddison (2009).

We begin the analysis by testing for cross-sectional dependence among the series of the panel. To this end, we apply the CD statistic of Pesaran (2004) to the panel of OECD countries. One advantage of this test is its robustness to single or multiple structural breaks, making it desirable in the empirical work. The calculated value of the CD test is 20.099, which indicate that we can reject the null hypothesis of no cross-sectional dependency at any acceptable level of significance.

Next, we apply the M-class unit root tests with two structural breaks in trend to each country individually we show the results in Table 9. The second and third column represent the break dates for each country while the last three columns present the individual M-class statistics. After looking at the break dates, we can see that the World War II period is the most frequent time of structural breaks. This is consistent with the previous studies on the OECD countries like Dawson and Strazicich (2010). We simulate the asymptotic critical values for the MSB^{GLS}, MZ_α^{GLS} and MZ_t^{GLS} tests and for each pair of λ. Although we do not present the critical values in the paper, they are available upon request. The MSB^{GLS} statistic suggests the rejection of the null hypothesis at the 5% significance level for Denmark, New Zealand and US and at the 10% level for Germany and UK. The other two statistics, MZ_α^{GLS} and MZ_t^{GLS} show similar results. Both tests suggest a rejection at the 5% level for Denmark, New Zealand and US and at the 10% level for UK. Overall, for the majority of countries we cannot reject the unit root hypothesis in favor of stationarity.

Finally, we apply the panel unit root statistics and we show the results at the bottom of Table 9. All panel unit root statistics are able to reject the non-stationary null hypothesis at 10% and two-thirds of them are able to reject the null at 5%. Since the rejection of the null hypothesis indicates that the respective test has good power, we can infer that all the panel statistics have good properties and indicate that the panel as a whole is stationary.

8 Conclusions

In this paper, we propose several panel data unit root tests that allow for multiple structural breaks and common factors to control for the presence of cross-section dependence. The test statistics are based on the use of GLS detrending procedure and the structural breaks are allowed under both the null and the alternative hypotheses. The model specification considers two cases according to the heterogeneity degree of the break points. First, we specify a mild heterogeneous framework where the break points are allowed to differ across individuals. Second, we also cover the case of homogeneous break points. The paper derives the limiting distribution of the individual unit root test statistics for the idiosyncratic disturbance term and the common factors. Further, we also show that panel data unit root test statistics can be defined through the combination of the individual test statistics of the idiosyncratic component.

The performance of the statistics that have been proposed is evaluated using a Monte Carlo simulation experiment. The simulations show that the test statistics perform well for the cases of known structural breaks. When the location of the structural breaks is not known *a priori* the panel statistics suffer for under-rejection when the time series dimension is large. Finally, we apply the proposed tests to a panel data set of annual real per capita GDP over the period 1870-2008 for 19 OECD developed countries. All panel statistics rejected the null hypothesis of panel data unit root in favor of I(0) stationarity for the idiosyncratic component of the real per capita GDP.

A Appendix

A.1 Proof of Theorem 1

The proof focuses on Model II, since it is the most general model specification that we consider. The proof for the other models follows this one. The GLS detrended variable can be written as:

$$\begin{aligned}
\tilde{y}_i &= y_i - z_i\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) y_i^{\bar{\alpha}} \\
&= y_i - z_i\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right)\left(y_i - \bar{\alpha}_i y_{i,-1}\right) \\
&= y_i - A_i\left(y_i - \bar{\alpha} y_{i,-1}\right) \\
&= F\delta_i - A_i F^{\bar{\alpha}}\delta_i + e_i - A_i e_i^{\bar{\alpha}},
\end{aligned}$$

where $A_i = z_i\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right)$ and $D_T = diag\{D_{1,T}, D_{2,T}\}$ $= diag(1, ..., 1, T^{-1/2}, ..., T^{-1/2})$. For subsequent developments, we define the partitioned vector of regressors $z_{i,t}\left(\lambda_i^0\right)$ as $z_{i,t}\left(\lambda_i^0\right) = \left(z_{i,t,1}'\left(\lambda_i^0\right), z_{i,t,2}'\left(\lambda_i^0\right)\right)'$. The term $z_{i,t,1}\left(\lambda_i^0\right)$ captures the $m+1$ regressors corresponding to the constant and the impulse dummy variables, while $z_{i,t,2}\left(\lambda_i^0\right)$ collects the $m+1$ trending regressors. Further, we also define $z_{i,t}^{\bar{\alpha}}\left(\lambda_i^0\right) = (z_{i,t,1}^{\bar{\alpha}\prime}\left(\lambda_i^0\right), z_{i,t,2}^{\bar{\alpha}\prime}\left(\lambda_i^0\right))'$ as the quasi-differenced $z_{i,t}\left(\lambda_i^0\right)$.

Taking the first difference we obtain the usual common factor representation

$$x_i = f\delta_i + \xi_i,$$

where $x_i = \Delta\tilde{y}_i$, $f = \Delta F - \Delta z_i\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) F^{\bar{\alpha}}\delta_i$ and $\xi_i = \Delta e_i - \Delta z_i\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) e_i^{\bar{\alpha}}$.

We can apply the method of principal components as in Bai and Ng (2004) and estimate f_t, δ_i and $\xi_{i,t}$ that are used to construct the unit root statistic which is based on the cumulative sum of the residuals

$$\tilde{e}_{i,t} = \sum_{j=2}^{t} \tilde{\xi}_{i,j}$$

Let us first focus on the idiosyncratic component $\xi_{i,t}$. Substracting $x_{i,t} = \tilde{f}_t'\tilde{\delta}_i + \tilde{\xi}_{i,t}$ from $x_{i,t} = f_t'\delta_i + \xi_{i,t}$ yields

$$\tilde{\xi}_{i,t} = \xi_{i,t} + f_t'\delta_i - \tilde{f}_t'\tilde{\delta}_i.$$

Following Bai and Ng (2004) and Bai and Carrion-Silvestre (2009), we can rewrite this equation as

$$\begin{aligned}
\tilde{\xi}_{i,t} &= \xi_{i,t} + f_t' H H^{-1}\delta_i - \tilde{f}_t' H^{-1}\delta_i + \tilde{f}_t' H^{-1}\delta_i - \tilde{f}_t'\tilde{\delta}_i \\
&= \xi_{i,t} + \left(H' f_t - \tilde{f}_t\right)' H^{-1}\delta_i - \tilde{f}_t'\left(\tilde{\delta}_i - H^{-1}\delta_i\right) \\
&= \xi_{i,t} - \eta_t' H^{-1}\delta_i - \tilde{f}_t'\kappa_i, \qquad (20)
\end{aligned}$$

where $\eta_t = \left(\tilde{f}_t - H' f_t\right)$ and $\kappa_i = \left(\tilde{\delta}_i - H^{-1}\delta_i\right)$.

Using Lemma 3 and C1 from Bai and Ng (2004) and Theorem 2 from Bai and Carrion-i-Silvestre (2009), with the condition that $N, T \to \infty$, we obtain

$$
\begin{aligned}
T^{-1/2}\left\|\sum_{j=2}^{t}\eta_j' H^{-1}\delta_i\right\| &= O_p\left(C_{NT}^{-1}\right) \\
T^{-1/2}\left\|\sum_{j=2}^{t}\tilde{f}_j\kappa_i\right\| &= O_p\left(C_{NT}^{-1}\right),
\end{aligned}
$$

where $C_{NT} = \min\left[\sqrt{N}, \sqrt{T}\right]$.

We define the cumulative sum residuals as $\tilde{e}_{i,t} = \sum_{j=2}^{t}\tilde{\xi}_{i,j}$ and rewrite the previous equation as

$$
T^{-1/2}\tilde{e}_{i,t} = T^{-1/2}\sum_{j=2}^{t}\xi_{i,j} + O_p\left(C_{NT}^{-1}\right).
$$

Note that $\xi_{i,t} = \Delta e_{i,t} - \Delta z_{i,t}\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) e_i^{\bar{\alpha}}$. Thus,

$$
\begin{aligned}
T^{-1/2}\sum_{j=2}^{t}\xi_{i,j} &= T^{-1/2}\sum_{j=2}^{t}\Delta e_{i,j} - T^{-1/2}\sum_{j=2}^{t}\Big[\Delta z_{i,t}\left(\lambda_i^0\right) \\
&\quad D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) e_i^{\bar{\alpha}}\Big] \\
&= T^{-1/2} e_{i,t} - T^{-1/2} e_{i,1} \\
&\quad -T^{-1/2} z_{i,t}\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) e_i^{\bar{\alpha}} \\
&\quad +T^{-1/2} z_{i,1}\left(\lambda_i^0\right) D_{i,T}\left(D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_i^{\bar{\alpha}}\left(\lambda_i^0\right) D_{i,T}\right)^{-1} D_{i,T} z_i^{\bar{\alpha}\prime}\left(\lambda_i^0\right) e_i^{\bar{\alpha}}.
\end{aligned}
$$

The second terms is $o_p(1)$ since $T^{-1/2} e_{i,1} \xrightarrow{p} 0$. Further, $T^{-1/2} z_{i,t}\left(\lambda_i^0\right)\left(z_{i,1}^{\bar{\alpha}\prime}\left(\lambda_i^0\right) z_{i,1}^{\bar{\alpha}}\left(\lambda_i^0\right)\right)^{-1}$ $z_{i,1}^{\bar{\alpha}\prime}\left(\lambda_i^0\right) e_i^{\bar{\alpha}} \xrightarrow{p} 0$ $\forall t$ and $T^{-1/2} D_{i,T,2} z_{i,2}^{\bar{\alpha}}\left(\lambda_i^0\right) \to z_2(r)$ uniformly in $r \in [0,1]$, where $z_2(s)$ $= \left(s, \left(s-\lambda_1^0\right) 1\left(s>\lambda_1^0\right), \ldots, \left(s-\lambda_m^0\right) 1\left(s>\lambda_m^0\right)\right)'$. Taking into account these elements, we can see that as $N, T \to \infty$ with $N/T \to 0$, we obtain

$$
T^{-1/2}\sum_{j=2}^{t}\xi_{i,j} \Rightarrow \sigma_i\left[W_{i,c}(s) - z_2(s) A\left(\lambda^0\right)^{-1}\bar{V}_i\left(\lambda^0\right)\right],
$$

where

$$
A(\lambda^0) = \begin{bmatrix}
a\left(\lambda_0^0, \lambda_0^0\right) & a\left(\lambda_0^0, \lambda_1^0\right) & \cdots & a\left(\lambda_0^0, \lambda_m^0\right) \\
 & a\left(\lambda_1^0, \lambda_1^0\right) & \cdots & a\left(\lambda_1^0, \lambda_m^0\right) \\
 & & \ddots & \vdots \\
 & & & a\left(\lambda_m^0, \lambda_m^0\right)
\end{bmatrix},
$$

with elements defined as

$$
a\left(\lambda_i^0, \lambda_j^0\right) = \frac{1}{6}\left(1-\lambda_j^0\right)\left[6\bar{c}\left(\lambda_i^0 - 1\right) + \bar{c}^2\left(\lambda_j^0\left(3\lambda_i^0 - 1\right) - 3\lambda_i^0 - \lambda_j^{0^2} + 2\right) + 6\right],
$$

with $\lambda_i^0 < \lambda_j^0$, $\forall i, j = 0, 1, \ldots, m$, where $\lambda_0^0 = 0$. Finally, $\bar{V}_i\left(\lambda^0\right) = \left(V_i\left(\lambda_0^0\right), \ldots, V_i\left(\lambda_m^0\right)\right)'$ with

$$\begin{aligned} V_i\left(\lambda_j^0\right) &= \left(1 + \lambda_j^0 \bar{c}\right)\left((W_i(1) - W_i(\lambda_j^0)) + (c - \bar{c}) \int_{\lambda_j^0}^1 W_{i,c}\left(s\right) ds\right) \\ &\quad -\bar{c} \int_{\lambda_j^0}^1 s dW_i\left(s\right) - (c - \bar{c})\, \bar{c} \int_{\lambda_j^0}^1 s W_{i,c}\left(s\right) ds. \end{aligned}$$

Consequently,

$$\begin{aligned} T^{-1/2} \tilde{e}_{i,t} &\Rightarrow \sigma_i \left[W_{i,c}\left(s\right) - z_2\left(s\right) A\left(\lambda^0\right)^{-1} \bar{V}_i\left(\lambda^0\right)\right] \\ &\equiv \sigma_i V_{i,c,\bar{c}}\left(s, \lambda^0\right), \end{aligned}$$

and by the Functional Central Limit Theorem (FCLT) we have that

$$MSB_i^{GLS} \Rightarrow \left(\int_0^1 V_{i,c,\bar{c}}\left(s, \lambda^0\right)^2 ds\right)^{1/2}.$$

The limit distribution of the other two test statistics follows easily from the developments above. Further, note that the limiting distribution for Model I is the same as the ones derived for Model II since the impulse dummies – i.e., the elements collected in $z_{i,1}^{\bar{\alpha}}$ – are asymptotically negligible, as shown above.

Let us now focus on the estimated common factors $\tilde{F}_t$. From $\tilde{f}_t = H f_t + v_t$ we have in the limit

$$\begin{aligned} \tilde{F}_t &= H \sum_{s=2}^{t} f_s + \sum_{s=2}^{t} v_s \\ &= H \sum_{s=2}^{t} (\Delta F_s - \Delta z_s\left(\lambda^0\right) D_T \left(D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right) z^{\bar{\alpha}}\left(\lambda^0\right) D_T\right)^{-1} \qquad (21) \\ &\quad D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right) F^{\bar{\alpha}} \bar{\delta}) + \sum_{s=2}^{t} v_s \\ &= H \Big[F_t - F_1 - z_t\left(\lambda^0\right) D_T \left(D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right) z^{\bar{\alpha}}\left(\lambda^0\right) D_T\right)^{-1} D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right) F^{\bar{\alpha}} \bar{\delta} \\ &\quad + z_1\left(\lambda^0\right) D_T \left(D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right) z^{\bar{\alpha}}\left(\lambda^0\right) D_T\right)^{-1} D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right) F^{\bar{\alpha}} \bar{\delta}\Big] \\ &\quad + V_t, \end{aligned}$$

where we have defined $z_t\left(\lambda^0\right)$ as the vector of regressors defined by the use of $T_j^0 = E\left(T_{i,j}^0\right) = N^{-1} \sum_{i=1}^N T_{i,j}^0$, $j = 0, 1, \ldots, m$. Further, note that in (21) we have $\bar{\delta}$, provided that it holds that

$$
\begin{aligned}
N^{-1}\sum_{i=1}^{N}\tilde{F}_t &= N^{-1}\sum_{i=1}^{N}\left[H\sum_{s=2}^{t}\left(\Delta F_s-\Delta z_s\left(\lambda^0\right)\right.\right.\\
&\qquad \left.\left.D_T\left(D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right)z^{\bar{\alpha}}\left(\lambda^0\right)D_T\right)^{-1}D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right)F^{\bar{\alpha}}\delta_i\right)+\sum_{s=2}^{t}v_s\right]\\
\tilde{F}_t &= H\sum_{s=2}^{t}\left(\Delta F_s-\Delta z_s\left(\lambda^0\right)D_T\left(D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right)z^{\bar{\alpha}}\left(\lambda^0\right)D_T\right)^{-1}D_T z^{\bar{\alpha}\prime}\left(\lambda^0\right)F^{\bar{\alpha}}\bar{\delta}\right)\\
&\quad +\sum_{s=2}^{t}v_s.
\end{aligned}
$$

Then, we can define $\tilde{F}_t^d = HF_t^d + V_t^d$, where the d superscript denotes that the variable has been detrended. In our case, the detrending is based on the use of the quasi-GLS procedure described above, so that we define $\tilde{F}_t^d = \tilde{F}_t - z_t'\left(\lambda^0\right)\hat{\psi}$ where $z_t\left(\lambda^0\right)$ is the vector of regressors defined by the use of $T_j^0 = E\left(T_{i,j}^0\right) = N^{-1}\sum_{i=1}^{N}T_{i,j}^0$ and $\hat{\psi}$ is the matrix of parameters that has been obtained using the GLS-detrending procedure. Using these elements and the developments above, we have

$$
\left(1/\sqrt{T}\right)\tilde{F}_t^d = H\left(1/\sqrt{T}\right)F_t^d + O_p\left(C_{NT}^{-1}\right),
$$

and the proof of the limiting distribution of the M-type unit root test statistics for the case of just one common factor ($r=1$) follows the one for the idiosyncratic component replacing $e_{i,t}$ by $\tilde{F}_t^d$.

When we have more than one common factor ($r>1$) we can proceed to apply the MQ tests in Bai and Ng (2004) in order to know how many I(1) and I(0) common factors we have. In this case, the limiting distribution of the MQ test statistics is given in Bai and Ng (2004) but using GLS-detrended-based Brownian motions functionals instead of OLS-detrended Brownian motions ones.

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Table 1: Asymptotic critical values for the $MQ(q, \lambda)$ tests for Model 0

r	1%	5%	10%
1	-13.78	-8.19	-5.82
2	-25.11	-18.16	-14.96
3	-35.27	-27.22	-23.53
4	-45.22	-36.31	-32.16
5	-54.26	-44.87	-40.52
6	-63.65	-53.63	-48.92

The moments of the limiting distribution of the statistics by means of Monte Carlo simulation, using 1,000 steps to approximate the Brownian motion functionals and 100,000 replications.

Table 2: Asymptotic critical values for the $MQ(q, \lambda)$ tests for Models I and II, one structural break case

	$\lambda^0 = 0.1$			$\lambda^0 = 0.2$			$\lambda^0 = 0.3$		
r	1%	5%	10%	1%	5%	10%	1%	5%	10%
1	-30.12	-22.28	-18.69	-31.11	-23.44	-19.81	-31.92	-23.83	-20.29
2	-41.08	-32.43	-28.30	-41.76	-33.26	-29.34	-42.09	-33.69	-29.67
3	-50.84	-41.48	-37.12	-51.51	-42.24	-37.93	-51.79	-42.75	-38.35
4	-59.87	-50.08	-45.56	-60.37	-50.99	-46.37	-60.98	-51.24	-46.63
5	-68.92	-58.65	-53.62	-69.45	-59.41	-54.49	-69.82	-59.66	-54.82
6	-77.35	-67.03	-61.90	-78.28	-67.54	-62.61	-78.30	-67.80	-62.77
	$\lambda^0 = 0.4$			$\lambda^0 = 0.5$			$\lambda^0 = 0.6$		
r	1%	5%	10%	1%	5%	10%	1%	5%	10%
1	-31.60	-23.97	-20.32	-31.85	-23.86	-20.25	-31.11	-23.23	-19.63
2	-42.07	-33.70	-29.72	-42.20	-33.66	-29.59	-41.90	-33.26	-29.25
3	-51.58	-42.57	-38.24	-51.52	-42.54	-38.16	-51.24	-42.14	-37.85
4	-60.54	-51.17	-46.63	-60.91	-51.11	-46.41	-60.32	-50.69	-46.17
5	-69.72	-59.49	-54.70	-69.09	-59.36	-54.53	-68.90	-59.11	-54.40
6	-78.25	-67.67	-62.68	-77.92	-67.61	-62.62	-77.65	-67.29	-62.33
	$\lambda^0 = 0.7$			$\lambda^0 = 0.8$			$\lambda^0 = 0.9$		
r	1%	5%	10%	1%	5%	10%	1%	5%	10%
1	-30.11	-22.29	-18.75	-28.90	-21.14	-17.67	-26.80	-19.38	-16.01
2	-41.07	-32.58	-28.62	-40.02	-31.49	-27.59	-38.01	-29.59	-25.70
3	-50.51	-41.50	-37.15	-50.00	-40.70	-36.50	-48.28	-38.96	-34.64
4	-60.02	-50.40	-45.84	-59.04	-49.52	-44.97	-57.40	-47.80	-43.28
5	-68.98	-58.98	-54.00	-67.99	-58.17	-53.25	-66.78	-56.42	-51.63
6	-77.38	-67.13	-61.96	-76.77	-66.38	-61.45	-75.31	-65.13	-60.10

The moments of the limiting distribution of the statistics by means of Monte Carlo simulation, using 1,000 steps to approximate the Brownian motion functionals and 100,000 replications.

Table 3: Mean and variance for the $M-class$ statistics

1 KNOWN BREAK

	MSB^{GLS}		MZ_{α}^{GLS}		MZ_{t}^{GLS}	
T	Mean	Variance	Mean	Variance	Mean	Variance
50	0.261456	0.005521		-8.282115	16.489818	
100	0.239627	0.004043		-9.935691	24.929351	
200	0.230758	0.003658		-10.823839	30.655044	
	0.227108	0.003485		-11.184445	34.052021	

2 KNOWN BREAKS

	MSB^{GLS}		MZ_{α}^{GLS}		MZ_{t}^{GLS}	
T	Mean	Variance	Mean	Variance	Mean	Variance
50	0.216900	0.002273		-11.538986	18.016178	
100	0.199408	0.001760		-13.776592	28.413185	
200	0.190975	0.001673		-15.248692	38.824724	
	0.188562	0.001690		-15.643370	43.778604	

1 UNKNOWN BREAK

	MSB^{GLS}		MZ_{α}^{GLS}		MZ_{t}^{GLS}	
T	Mean	Variance	Mean	Variance	Mean	Variance
50	0.250518	0.004387		-8.656986	15.602375	
100	0.244937	0.004483		-9.508741	25.471744	
200	0.265740	0.015757		-9.702224	38.359919	
	0.288416	0.038645		-9.921242	42.732074	

2 UNKNOWN BREAKS

	MSB^{GLS}		MZ_{α}^{GLS}		MZ_{t}^{GLS}	
T	Mean	Variance	Mean	Variance	Mean	Variance
50	0.209529	0.002359		-12.260359	18.159997	
100	0.205955	0.002338		-13.132034	32.312824	
200	0.263366	0.027457		-12.152745	63.590714	
	0.317205	0.066611		-11.753740	78.783104	

Simulations are based on 1000 replications. These values are valid for the Monte Carlo simulation presented in Section 6. The DGP for the one break case is given by Equations (12) to (15). For the case on 1 known break, the values for the mean and the variance are calculated using $\lambda^0 = 0.5$. The DGP for the two breaks case is given by Equations (16) to (19). The values for the mean and the variance for the case of 2 known breaks are calculated using $\lambda_1^0 = 0.3$ and $\lambda_2^0 = 0.7$.

Table 4: Mean and variance for the M class statistics for 1 known break

MSB^{GLS}

	$\lambda^0 = 0.1$		$\lambda^0 = 0.2$		$\lambda^0 = 0.3$		$\lambda^0 = 0.4$		$\lambda^0 = 0.5$		$\lambda^0 = 0.6$		$\lambda^0 = 0.7$		$\lambda^0 = 0.8$		$\lambda^0 = 0.9$	
T	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
50	0.255	0.004	0.244	0.003	0.244	0.003	0.249	0.004	0.262	0.006	0.278	0.008	0.298	0.011	0.320	0.014	0.349	0.017
100	0.246	0.004	0.234	0.003	0.229	0.003	0.232	0.003	0.240	0.004	0.254	0.006	0.270	0.008	0.289	0.010	0.312	0.012
200	0.241	0.004	0.230	0.004	0.224	0.003	0.224	0.003	0.230	0.004	0.239	0.005	0.255	0.006	0.274	0.009	0.295	0.010

MZ_{α}^{GLS}

	$\lambda^0 = 0.1$		$\lambda^0 = 0.2$		$\lambda^0 = 0.3$		$\lambda^0 = 0.4$		$\lambda^0 = 0.5$		$\lambda^0 = 0.6$		$\lambda^0 = 0.7$		$\lambda^0 = 0.8$		$\lambda^0 = 0.9$	
T	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
50	-8.250	14.629	-8.702	14.064	-8.785	14.184	-8.617	15.507	-8.320	16.781	-7.854	17.549	-7.180	17.511	-6.469	16.467	-5.725	13.809
100	-9.291	23.069	-10.098	24.238	-10.278	23.797	-10.212	23.795	-9.848	24.086	-9.415	25.031	-8.728	24.994	-7.889	23.194	-6.965	19.601
200	-9.851	29.598	-10.709	31.300	-11.145	31.786	-11.203	31.043	-10.926	31.462	-10.364	30.749	-9.640	29.710	-8.907	29.388	-7.768	23.596

MZ_{t}^{GLS}

	$\lambda^0 = 0.1$		$\lambda^0 = 0.2$		$\lambda^0 = 0.3$		$\lambda^0 = 0.4$		$\lambda^0 = 0.5$		$\lambda^0 = 0.6$		$\lambda^0 = 0.7$		$\lambda^0 = 0.8$		$\lambda^0 = 0.9$	
T	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
50	-1.887	0.264	-1.962	0.250	-1.971	0.245	-1.943	0.264	-1.897	0.304	-1.842	0.318	-1.757	0.342	-1.670	0.338	-1.576	0.304
100	-2.005	0.360	-2.102	0.343	-2.120	0.330	-2.125	0.324	-2.095	0.331	-2.039	0.356	-1.954	0.368	-1.866	0.364	-1.774	0.337
200	-2.069	0.414	-2.168	0.411	-2.213	0.373	-2.225	0.374	-2.195	0.363	-2.161	0.374	-2.082	0.392	-1.985	0.381	-1.865	0.354

The mean and the variance for each break are simulated using on 10,000 replications.

Table 5: Empirical size of panel unit root statistics for 1 known break for $N = 20$ and $\lambda^0 = 0.5$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.044	0.054	0.048	0.045	0.047	0.044	0.053	0.055	0.058	0.050	0.053	0.053
		0.95	0.048	0.061	0.065	0.055	0.064	0.059	0.069	0.070	0.066	0.057	0.060	0.061
		1	0.052	0.064	0.059	0.051	0.058	0.060	0.064	0.077	0.076	0.062	0.062	0.062
	1	0.9	0.032	0.038	0.034	0.045	0.045	0.044	0.054	0.060	0.053	0.032	0.036	0.038
		0.95	0.052	0.069	0.065	0.055	0.056	0.052	0.071	0.069	0.065	0.066	0.072	0.065
		1	0.046	0.054	0.050	0.037	0.045	0.048	0.051	0.057	0.060	0.045	0.048	0.049
	10	0.9	0.049	0.061	0.053	0.057	0.051	0.046	0.066	0.062	0.057	0.059	0.060	0.055
		0.95	0.043	0.078	0.064	0.060	0.067	0.061	0.074	0.079	0.076	0.063	0.069	0.067
		1	0.041	0.060	0.055	0.065	0.066	0.059	0.075	0.074	0.073	0.054	0.054	0.057
100	0.5	0.9	0.040	0.050	0.050	0.048	0.041	0.048	0.053	0.050	0.065	0.046	0.042	0.049
		0.95	0.047	0.050	0.058	0.045	0.042	0.050	0.059	0.048	0.060	0.052	0.049	0.056
		1	0.051	0.048	0.056	0.045	0.042	0.044	0.056	0.049	0.055	0.055	0.053	0.055
	1	0.9	0.031	0.049	0.052	0.055	0.039	0.054	0.064	0.059	0.066	0.048	0.048	0.050
		0.95	0.043	0.049	0.057	0.050	0.044	0.054	0.055	0.056	0.063	0.051	0.048	0.056
		1	0.038	0.053	0.043	0.054	0.052	0.065	0.067	0.065	0.073	0.054	0.043	0.041
	10	0.9	0.046	0.051	0.055	0.056	0.050	0.057	0.063	0.059	0.067	0.052	0.052	0.057
		0.95	0.051	0.055	0.059	0.056	0.041	0.051	0.061	0.055	0.062	0.056	0.049	0.057
		1	0.045	0.049	0.050	0.039	0.039	0.053	0.055	0.049	0.064	0.049	0.047	0.049
200	0.5	0.9	0.046	0.058	0.053	0.040	0.044	0.038	0.051	0.054	0.043	0.054	0.059	0.051
		0.95	0.066	0.080	0.069	0.063	0.064	0.055	0.070	0.078	0.061	0.075	0.076	0.069
		1	0.043	0.054	0.039	0.045	0.051	0.039	0.060	0.066	0.049	0.051	0.051	0.039
	1	0.9	0.049	0.064	0.050	0.051	0.054	0.042	0.065	0.063	0.048	0.056	0.056	0.043
		0.95	0.059	0.067	0.054	0.046	0.053	0.042	0.063	0.065	0.051	0.063	0.062	0.053
		1	0.055	0.074	0.058	0.067	0.069	0.056	0.080	0.087	0.070	0.065	0.066	0.057
	10	0.9	0.056	0.064	0.051	0.055	0.056	0.047	0.067	0.068	0.056	0.064	0.057	0.050
		0.95	0.049	0.063	0.048	0.050	0.053	0.045	0.061	0.063	0.054	0.056	0.055	0.048
		1	0.055	0.064	0.051	0.052	0.053	0.043	0.061	0.069	0.054	0.056	0.058	0.050

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 6: Empirical size of panel unit root statistics for 2 known breaks for $N = 20$, $\lambda_1^0 = 0.3$ and $\lambda_2^0 = 0.7$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.043	0.046	0.045	0.047	0.045	0.049	0.059	0.057	0.058	0.050	0.042	0.048
		0.95	0.054	0.058	0.067	0.062	0.056	0.055	0.070	0.064	0.066	0.063	0.062	0.070
		1	0.042	0.055	0.056	0.054	0.047	0.042	0.067	0.057	0.059	0.059	0.053	0.057
	1	0.9	0.038	0.057	0.053	0.048	0.048	0.050	0.059	0.055	0.062	0.051	0.054	0.056
		0.95	0.044	0.051	0.052	0.058	0.057	0.057	0.075	0.069	0.068	0.057	0.049	0.054
		1	0.055	0.061	0.062	0.056	0.055	0.054	0.072	0.067	0.065	0.065	0.060	0.062
	10	0.9	0.039	0.053	0.059	0.052	0.050	0.056	0.062	0.060	0.071	0.051	0.052	0.061
		0.95	0.046	0.059	0.065	0.047	0.047	0.053	0.056	0.058	0.061	0.056	0.058	0.062
		1	0.039	0.048	0.060	0.049	0.047	0.045	0.063	0.060	0.060	0.050	0.050	0.057
100	0.5	0.9	0.053	0.066	0.048	0.054	0.062	0.045	0.066	0.072	0.058	0.056	0.058	0.047
		0.95	0.075	0.082	0.066	0.057	0.059	0.050	0.067	0.071	0.055	0.075	0.079	0.065
		1	0.050	0.067	0.050	0.059	0.065	0.045	0.071	0.073	0.060	0.056	0.059	0.050
	1	0.9	0.055	0.078	0.050	0.059	0.072	0.051	0.076	0.081	0.066	0.067	0.066	0.048
		0.95	0.056	0.062	0.045	0.049	0.052	0.038	0.056	0.061	0.050	0.057	0.061	0.044
		1	0.044	0.062	0.043	0.063	0.062	0.044	0.072	0.072	0.053	0.055	0.056	0.043
	10	0.9	0.060	0.067	0.053	0.063	0.067	0.050	0.075	0.075	0.061	0.060	0.062	0.052
		0.95	0.054	0.073	0.052	0.059	0.065	0.051	0.072	0.077	0.059	0.059	0.063	0.051
		1	0.055	0.051	0.043	0.045	0.051	0.040	0.055	0.054	0.049	0.050	0.050	0.042
200	0.5	0.9	0.049	0.057	0.050	0.046	0.054	0.051	0.064	0.072	0.060	0.053	0.051	0.048
		0.95	0.047	0.061	0.044	0.049	0.048	0.046	0.056	0.059	0.056	0.056	0.053	0.044
		1	0.046	0.056	0.048	0.044	0.046	0.041	0.054	0.059	0.052	0.057	0.053	0.046
	1	0.9	0.046	0.060	0.054	0.049	0.054	0.049	0.059	0.063	0.059	0.059	0.057	0.051
		0.95	0.050	0.053	0.047	0.049	0.056	0.050	0.056	0.064	0.056	0.053	0.050	0.049
		1	0.071	0.086	0.069	0.062	0.071	0.063	0.073	0.080	0.075	0.073	0.075	0.066
	10	0.9	0.055	0.063	0.055	0.049	0.054	0.053	0.063	0.071	0.062	0.061	0.058	0.051
		0.95	0.050	0.060	0.048	0.053	0.058	0.050	0.065	0.065	0.062	0.052	0.050	0.045
		1	0.050	0.054	0.046	0.052	0.057	0.052	0.061	0.063	0.058	0.052	0.049	0.043

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (16) through (19).

Table 7: Empirical size of panel unit root statistics for 1 unknown break for $N = 20$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.040	0.069	0.059	0.044	0.056	0.053	0.053	0.064	0.058	0.049	0.065	0.058
		0.95	0.053	0.083	0.074	0.062	0.075	0.064	0.066	0.084	0.074	0.065	0.082	0.070
		1	0.041	0.077	0.066	0.049	0.069	0.056	0.066	0.077	0.067	0.052	0.075	0.064
	1	0.9	0.043	0.065	0.052	0.045	0.062	0.055	0.052	0.067	0.062	0.054	0.063	0.054
		0.95	0.048	0.079	0.068	0.063	0.075	0.068	0.068	0.082	0.076	0.065	0.076	0.070
		1	0.051	0.092	0.074	0.066	0.082	0.071	0.073	0.093	0.085	0.066	0.082	0.075
	10	0.9	0.137	0.196	0.175	0.180	0.198	0.192	0.194	0.206	0.198	0.166	0.186	0.176
		0.95	0.138	0.196	0.181	0.180	0.205	0.198	0.189	0.217	0.207	0.165	0.186	0.185
		1	0.134	0.195	0.184	0.173	0.195	0.191	0.181	0.200	0.199	0.158	0.185	0.188
100	0.5	0.9	0.044	0.049	0.064	0.033	0.036	0.047	0.044	0.044	0.055	0.043	0.046	0.061
		0.95	0.052	0.057	0.071	0.050	0.050	0.059	0.054	0.056	0.062	0.058	0.064	0.070
		1	0.044	0.049	0.058	0.034	0.033	0.040	0.039	0.039	0.047	0.040	0.042	0.058
	1	0.9	0.048	0.040	0.058	0.034	0.035	0.040	0.041	0.040	0.046	0.038	0.042	0.057
		0.95	0.048	0.048	0.064	0.038	0.040	0.052	0.044	0.050	0.057	0.045	0.049	0.063
		1	0.038	0.044	0.055	0.033	0.034	0.040	0.045	0.040	0.049	0.040	0.043	0.055
	10	0.9	0.057	0.078	0.095	0.064	0.068	0.075	0.072	0.074	0.085	0.065	0.081	0.093
		0.95	0.093	0.105	0.122	0.092	0.093	0.100	0.096	0.097	0.111	0.100	0.106	0.122
		1	0.101	0.122	0.137	0.108	0.115	0.125	0.113	0.123	0.132	0.115	0.123	0.136
200	0.5	0.9	0.044	0.038	0.049	0.024	0.024	0.025	0.025	0.027	0.026	0.043	0.045	0.047
		0.95	0.041	0.052	0.060	0.034	0.033	0.030	0.041	0.038	0.037	0.054	0.054	0.053
		1	0.037	0.040	0.045	0.030	0.030	0.029	0.033	0.033	0.035	0.041	0.041	0.042
	1	0.9	0.041	0.040	0.045	0.023	0.026	0.026	0.030	0.029	0.029	0.044	0.045	0.043
		0.95	0.049	0.037	0.054	0.025	0.027	0.026	0.031	0.030	0.030	0.051	0.052	0.051
		1	0.037	0.040	0.042	0.024	0.023	0.022	0.028	0.025	0.026	0.042	0.042	0.039
	10	0.9	0.052	0.050	0.066	0.030	0.029	0.032	0.035	0.036	0.037	0.053	0.062	0.062
		0.95	0.047	0.050	0.055	0.024	0.028	0.030	0.034	0.034	0.035	0.051	0.054	0.054
		1	0.078	0.068	0.075	0.052	0.053	0.053	0.059	0.058	0.059	0.073	0.072	0.069

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 8: Empirical size of panel unit root statistics for 2 unknown breaks for $N = 20$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}			
50	0.5	0.9	0.022	0.038	0.033	0.035	0.034	0.039	0.043	0.041	0.044	0.037	0.037	0.037
		0.95	0.027	0.040	0.033	0.029	0.027	0.030	0.039	0.032	0.041	0.038	0.039	0.035
		1	0.020	0.035	0.033	0.025	0.028	0.036	0.033	0.035	0.043	0.032	0.033	0.039
	1	0.9	0.025	0.039	0.033	0.031	0.030	0.033	0.036	0.036	0.040	0.039	0.037	0.030
		0.95	0.022	0.043	0.043	0.046	0.045	0.045	0.051	0.049	0.051	0.042	0.043	0.044
		1	0.020	0.035	0.031	0.033	0.033	0.037	0.037	0.041	0.042	0.037	0.036	0.037
	10	0.9	0.087	0.112	0.114	0.114	0.119	0.131	0.123	0.129	0.136	0.108	0.114	0.120
		0.95	0.086	0.108	0.105	0.123	0.128	0.133	0.129	0.131	0.137	0.110	0.113	0.116
		1	0.079	0.117	0.109	0.122	0.131	0.137	0.130	0.137	0.140	0.114	0.116	0.123
100	0.5	0.9	0.047	0.043	0.062	0.043	0.029	0.039	0.046	0.040	0.049	0.051	0.044	0.060
		0.95	0.043	0.046	0.052	0.040	0.034	0.040	0.049	0.039	0.047	0.053	0.048	0.051
		1	0.039	0.046	0.059	0.045	0.035	0.040	0.048	0.043	0.050	0.055	0.047	0.061
	1	0.9	0.041	0.037	0.048	0.032	0.027	0.033	0.041	0.031	0.038	0.043	0.039	0.045
		0.95	0.036	0.034	0.044	0.032	0.023	0.025	0.036	0.027	0.034	0.038	0.035	0.043
		1	0.033	0.032	0.040	0.028	0.021	0.029	0.031	0.025	0.035	0.032	0.032	0.040
	10	0.9	0.061	0.064	0.072	0.060	0.053	0.062	0.066	0.060	0.070	0.072	0.068	0.072
		0.95	0.071	0.076	0.079	0.075	0.073	0.076	0.081	0.076	0.079	0.079	0.075	0.080
		1	0.062	0.075	0.079	0.078	0.072	0.076	0.086	0.077	0.082	0.079	0.074	0.080
200	0.5	0.9	0.099	0.042	0.069	0.024	0.022	0.023	0.031	0.025	0.027	0.057	0.047	0.046
		0.95	0.083	0.029	0.054	0.017	0.014	0.014	0.022	0.017	0.020	0.049	0.038	0.034
		1	0.092	0.048	0.057	0.029	0.023	0.024	0.033	0.029	0.029	0.058	0.051	0.052
	1	0.9	0.078	0.032	0.055	0.024	0.019	0.019	0.026	0.021	0.022	0.045	0.034	0.034
		0.95	0.079	0.032	0.052	0.017	0.012	0.010	0.019	0.017	0.018	0.044	0.036	0.035
		1	0.107	0.040	0.070	0.025	0.024	0.023	0.029	0.027	0.027	0.053	0.044	0.046
	10	0.9	0.082	0.048	0.060	0.031	0.026	0.024	0.034	0.030	0.029	0.058	0.050	0.051
		0.95	0.061	0.031	0.044	0.020	0.017	0.017	0.027	0.023	0.022	0.044	0.036	0.032
		1	0.059	0.034	0.044	0.022	0.019	0.019	0.023	0.022	0.021	0.042	0.037	0.037

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (16) through (19).

Table 9: Individual unit root tests and the two break dates

Country	$\tilde{T}_1$	$\tilde{T}_2$	λ used for cv	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
Australia	1889	1931	(0.2, 0.5)	0.170	-17.130	-2.910
Austria	1913	1947	(0.3, 0.6)	0.165	-17.607	-2.913
Belgium	1918	1943	(0.4, 0.6)	0.200	-12.370	-2.472
Canada	1917	1933	(0.4, 0.5)	0.192	-13.105	-2.509
Denmark	1939	1973	(0.5, 0.8)	0.147**	-23.219**	-3.402**
Finland	1892	1918	(0.2, 0.4)	0.194	-13.342	-2.583
France	1929	1945	(0.4, 0.6)	0.165	-17.122	-2.830
Germany	1913	1944	(0.3, 0.6)	0.152*	-20.914	-3.183
Italy	1918	1945	(0.4, 0.6)	0.295	-4.447	-1.312
Japan	1950	1973	(0.6, 0.8)	0.258	-6.836	-1.763
Netherlands	1918	1945	(0.4, 0.6)	0.168	-17.679	-2.972
New Zealand	1920	1934	(0.4, 0.5)	0.145**	-23.637**	-3.436**
Norway	1916	1939	(0.4, 0.5)	0.161	-17.394	-2.802
Portugal	1936	1961	(0.5, 0.7)	0.203	-10.688	-2.167
Spain	1935	1960	(0.5, 0.7)	0.265	-7.142	-1.890
Sweden	1916	1939	(0.4, 0.5)	0.229	-9.097	-2.083
Switzerland	1907	1944	(0.3, 0.6)	0.184	-14.497	-2.668
UK	1918	1943	(0.4, 0.6)	0.150*	-21.756*	-3.254*
USA	1929	1944	(0.4, 0.6)	0.143**	-24.572**	-3.505**

$\tilde{T}_1$ and $\tilde{T}_2$ represent the break dates and cv denotes the critical value; * and ** denote rejection of the null hypothesis at the 10% and 5% level of significance, respectively.

Table 10: Panel unit root tests (two breaks)

Panel test	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
MSB_m	-1.5616*	-1.7372**	-1.6879**
P	53.4213**	51.9885*	52.7378*
P_m	1.7689**	1.6046*	1.6905**
Z	-1.7764**	-1.6246*	-1.6712**

Note: * and ** denote rejection of the null hypothesis at the 10% and 5% level of significance, respectively.

Table 11: Empirical size of panel unit root statistics for 1 known break for $N = 20$ and $\lambda^0 = 0.3$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.043	0.057	0.043	0.037	0.046	0.047	0.040	0.063	0.065	0.042	0.045	0.048
		0.95	0.058	0.067	0.053	0.039	0.056	0.049	0.063	0.069	0.066	0.059	0.057	0.056
		1	0.061	0.074	0.067	0.066	0.074	0.066	0.077	0.088	0.080	0.064	0.072	0.066
	1	0.9	0.043	0.057	0.042	0.048	0.052	0.055	0.056	0.063	0.063	0.050	0.054	0.044
		0.95	0.060	0.067	0.059	0.057	0.059	0.053	0.064	0.067	0.064	0.060	0.067	0.055
		1	0.059	0.078	0.061	0.067	0.074	0.067	0.078	0.088	0.083	0.066	0.068	0.060
	10	0.9	0.050	0.063	0.054	0.051	0.054	0.050	0.060	0.064	0.060	0.056	0.059	0.057
		0.95	0.063	0.072	0.066	0.048	0.065	0.063	0.062	0.077	0.074	0.070	0.075	0.070
		1	0.058	0.073	0.067	0.060	0.061	0.052	0.072	0.073	0.063	0.061	0.068	0.064
100	0.5	0.9	0.051	0.055	0.048	0.041	0.052	0.052	0.050	0.059	0.059	0.054	0.055	0.048
		0.95	0.049	0.058	0.054	0.044	0.051	0.055	0.055	0.063	0.068	0.050	0.053	0.052
		1	0.049	0.060	0.054	0.039	0.046	0.048	0.044	0.053	0.054	0.053	0.055	0.053
	1	0.9	0.040	0.053	0.045	0.041	0.045	0.046	0.044	0.051	0.060	0.046	0.049	0.045
		0.95	0.043	0.064	0.060	0.045	0.052	0.056	0.056	0.068	0.067	0.047	0.062	0.060
		1	0.045	0.061	0.048	0.040	0.048	0.054	0.050	0.064	0.067	0.046	0.051	0.048
	10	0.9	0.050	0.069	0.052	0.054	0.063	0.063	0.064	0.077	0.075	0.049	0.059	0.053
		0.95	0.035	0.053	0.046	0.038	0.048	0.048	0.049	0.060	0.060	0.043	0.049	0.046
		1	0.055	0.074	0.066	0.041	0.055	0.056	0.051	0.067	0.069	0.054	0.068	0.069
200	0.5	0.9	0.043	0.070	0.054	0.054	0.057	0.048	0.067	0.071	0.054	0.056	0.059	0.053
		0.95	0.047	0.060	0.044	0.049	0.051	0.040	0.063	0.061	0.050	0.052	0.050	0.040
		1	0.055	0.074	0.057	0.056	0.058	0.049	0.066	0.073	0.058	0.064	0.072	0.054
	1	0.9	0.043	0.064	0.047	0.047	0.056	0.049	0.066	0.069	0.056	0.051	0.054	0.043
		0.95	0.044	0.071	0.054	0.064	0.065	0.048	0.073	0.075	0.064	0.065	0.063	0.051
		1	0.057	0.079	0.063	0.061	0.065	0.057	0.069	0.076	0.065	0.066	0.073	0.062
	10	0.9	0.059	0.083	0.061	0.068	0.073	0.054	0.086	0.088	0.072	0.067	0.077	0.062
		0.95	0.044	0.070	0.045	0.050	0.059	0.046	0.068	0.070	0.059	0.051	0.057	0.043
		1	0.053	0.077	0.061	0.067	0.069	0.060	0.073	0.081	0.069	0.056	0.060	0.056

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 12: Empirical size of panel unit root statistics for 1 known break for $N = 20$ and $\lambda^0 = 0.7$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.046	0.062	0.053	0.043	0.052	0.050	0.050	0.068	0.063	0.056	0.061	0.057
		0.95	0.056	0.066	0.059	0.044	0.061	0.046	0.061	0.071	0.060	0.062	0.066	0.054
		1	0.056	0.071	0.058	0.052	0.067	0.053	0.059	0.075	0.068	0.060	0.069	0.061
	1	0.9	0.031	0.056	0.039	0.030	0.052	0.043	0.045	0.061	0.056	0.041	0.054	0.041
		0.95	0.050	0.065	0.056	0.035	0.052	0.044	0.047	0.067	0.056	0.055	0.063	0.053
		1	0.043	0.066	0.061	0.036	0.052	0.047	0.045	0.065	0.056	0.051	0.061	0.055
	10	0.9	0.052	0.076	0.055	0.051	0.065	0.056	0.059	0.079	0.073	0.061	0.065	0.057
		0.95	0.056	0.088	0.062	0.040	0.060	0.047	0.056	0.075	0.065	0.067	0.078	0.065
		1	0.046	0.079	0.061	0.056	0.071	0.067	0.072	0.083	0.077	0.064	0.070	0.059
100	0.5	0.9	0.041	0.056	0.064	0.047	0.047	0.058	0.052	0.057	0.074	0.052	0.055	0.064
		0.95	0.041	0.063	0.062	0.059	0.058	0.071	0.064	0.068	0.079	0.054	0.057	0.058
		1	0.053	0.061	0.070	0.049	0.053	0.065	0.057	0.063	0.071	0.060	0.063	0.067
	1	0.9	0.036	0.052	0.056	0.036	0.038	0.049	0.047	0.048	0.064	0.045	0.048	0.054
		0.95	0.038	0.064	0.058	0.049	0.054	0.065	0.061	0.063	0.078	0.047	0.049	0.055
		1	0.050	0.061	0.058	0.046	0.052	0.064	0.058	0.062	0.081	0.054	0.053	0.059
	10	0.9	0.044	0.061	0.059	0.047	0.051	0.062	0.057	0.064	0.075	0.053	0.054	0.057
		0.95	0.055	0.072	0.068	0.058	0.056	0.068	0.063	0.069	0.077	0.062	0.063	0.069
		1	0.049	0.064	0.069	0.049	0.048	0.066	0.061	0.069	0.083	0.059	0.063	0.070
200	0.5	0.9	0.037	0.066	0.061	0.043	0.059	0.050	0.057	0.074	0.061	0.049	0.064	0.056
		0.95	0.041	0.056	0.045	0.037	0.049	0.040	0.045	0.055	0.047	0.038	0.047	0.042
		1	0.046	0.086	0.068	0.066	0.078	0.067	0.076	0.091	0.079	0.058	0.076	0.064
	1	0.9	0.048	0.074	0.061	0.044	0.063	0.049	0.062	0.072	0.062	0.051	0.070	0.060
		0.95	0.035	0.056	0.042	0.041	0.056	0.045	0.053	0.067	0.054	0.038	0.046	0.039
		1	0.048	0.067	0.057	0.040	0.062	0.046	0.057	0.075	0.065	0.056	0.067	0.057
	10	0.9	0.050	0.078	0.063	0.054	0.064	0.054	0.063	0.076	0.069	0.057	0.072	0.059
		0.95	0.035	0.081	0.065	0.051	0.068	0.061	0.064	0.079	0.067	0.056	0.067	0.055
		1	0.034	0.076	0.060	0.054	0.068	0.058	0.063	0.081	0.068	0.053	0.068	0.054

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 13: Empirical size of panel unit root statistics for 1 known break for $N = 40$ and $\lambda^0 = 0.5$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.056	0.061	0.057	0.050	0.054	0.050	0.061	0.062	0.058	0.058	0.055	0.052
		0.95	0.055	0.067	0.063	0.059	0.067	0.064	0.071	0.075	0.068	0.066	0.067	0.069
		1	0.058	0.070	0.065	0.047	0.055	0.050	0.058	0.064	0.060	0.060	0.064	0.067
	1	0.9	0.031	0.051	0.044	0.049	0.053	0.051	0.058	0.062	0.056	0.043	0.044	0.043
		0.95	0.053	0.057	0.057	0.048	0.050	0.050	0.057	0.058	0.055	0.054	0.056	0.056
		1	0.059	0.070	0.069	0.063	0.065	0.060	0.071	0.075	0.065	0.061	0.068	0.071
	10	0.9	0.048	0.057	0.054	0.052	0.056	0.053	0.058	0.061	0.061	0.051	0.055	0.054
		0.95	0.053	0.056	0.061	0.050	0.051	0.049	0.058	0.059	0.053	0.054	0.060	0.062
		1	0.046	0.058	0.057	0.056	0.051	0.055	0.060	0.068	0.068	0.058	0.058	0.057
100	0.5	0.9	0.036	0.046	0.047	0.061	0.063	0.052	0.069	0.072	0.059	0.047	0.052	0.042
		0.95	0.051	0.054	0.060	0.064	0.066	0.056	0.071	0.072	0.063	0.064	0.065	0.053
		1	0.040	0.055	0.059	0.061	0.069	0.056	0.074	0.077	0.063	0.061	0.066	0.055
	1	0.9	0.043	0.046	0.051	0.057	0.062	0.050	0.066	0.073	0.060	0.052	0.064	0.049
		0.95	0.050	0.053	0.059	0.065	0.069	0.060	0.079	0.080	0.068	0.060	0.069	0.055
		1	0.056	0.060	0.066	0.076	0.077	0.062	0.085	0.087	0.071	0.071	0.074	0.058
	10	0.9	0.040	0.041	0.046	0.058	0.059	0.049	0.063	0.068	0.056	0.046	0.048	0.040
		0.95	0.047	0.070	0.071	0.078	0.079	0.063	0.081	0.085	0.074	0.071	0.083	0.065
		1	0.058	0.058	0.066	0.070	0.076	0.056	0.078	0.091	0.069	0.064	0.074	0.059
200	0.5	0.9	0.045	0.054	0.038	0.041	0.032	0.033	0.053	0.040	0.039	0.049	0.041	0.047
		0.95	0.041	0.047	0.039	0.044	0.037	0.042	0.047	0.041	0.043	0.049	0.040	0.047
		1	0.046	0.047	0.037	0.037	0.027	0.027	0.042	0.034	0.041	0.047	0.041	0.046
	1	0.9	0.055	0.061	0.055	0.046	0.041	0.045	0.055	0.045	0.056	0.053	0.052	0.064
		0.95	0.059	0.072	0.053	0.065	0.052	0.055	0.077	0.064	0.064	0.062	0.049	0.061
		1	0.055	0.074	0.054	0.067	0.055	0.061	0.076	0.062	0.070	0.063	0.052	0.062
	10	0.9	0.037	0.058	0.043	0.053	0.048	0.053	0.063	0.054	0.057	0.046	0.038	0.047
		0.95	0.055	0.053	0.035	0.048	0.043	0.041	0.056	0.048	0.051	0.054	0.041	0.045
		1	0.050	0.065	0.061	0.050	0.046	0.054	0.064	0.056	0.064	0.062	0.053	0.061

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 14: Empirical size of panel unit root statistics for 1 known break for $N = 60$ and $\lambda^0 = 0.5$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.045	0.051	0.050	0.045	0.051	0.049	0.051	0.059	0.057	0.050	0.052	0.051
		0.95	0.050	0.058	0.060	0.049	0.055	0.055	0.055	0.063	0.063	0.048	0.061	0.061
		1	0.054	0.056	0.056	0.048	0.052	0.046	0.056	0.059	0.056	0.060	0.058	0.055
	1	0.9	0.042	0.056	0.047	0.047	0.052	0.045	0.055	0.057	0.050	0.052	0.050	0.046
		0.95	0.051	0.071	0.062	0.061	0.061	0.055	0.064	0.064	0.060	0.063	0.065	0.058
		1	0.044	0.049	0.045	0.043	0.050	0.045	0.051	0.053	0.052	0.055	0.050	0.046
	10	0.9	0.032	0.042	0.038	0.039	0.047	0.041	0.046	0.054	0.047	0.036	0.037	0.040
		0.95	0.056	0.071	0.072	0.049	0.056	0.055	0.053	0.065	0.065	0.059	0.072	0.073
		1	0.052	0.066	0.068	0.046	0.053	0.056	0.054	0.062	0.064	0.054	0.062	0.066
100	0.5	0.9	0.042	0.037	0.043	0.050	0.051	0.039	0.054	0.063	0.045	0.044	0.050	0.040
		0.95	0.049	0.048	0.062	0.056	0.064	0.049	0.065	0.075	0.058	0.062	0.072	0.057
		1	0.054	0.051	0.061	0.052	0.055	0.043	0.057	0.063	0.049	0.061	0.075	0.056
	1	0.9	0.038	0.044	0.047	0.056	0.059	0.044	0.065	0.066	0.047	0.052	0.056	0.042
		0.95	0.048	0.049	0.055	0.055	0.064	0.043	0.068	0.075	0.051	0.061	0.068	0.051
		1	0.053	0.048	0.058	0.064	0.068	0.046	0.072	0.077	0.051	0.063	0.069	0.054
	10	0.9	0.043	0.035	0.050	0.048	0.050	0.038	0.054	0.059	0.044	0.053	0.056	0.046
		0.95	0.035	0.052	0.052	0.063	0.068	0.050	0.070	0.077	0.061	0.053	0.061	0.047
		1	0.055	0.059	0.067	0.067	0.072	0.056	0.074	0.085	0.063	0.072	0.083	0.058
200	0.5	0.9	0.051	0.062	0.046	0.052	0.040	0.046	0.060	0.048	0.054	0.059	0.045	0.055
		0.95	0.057	0.062	0.048	0.056	0.044	0.050	0.060	0.052	0.057	0.057	0.048	0.063
		1	0.066	0.072	0.049	0.062	0.053	0.058	0.071	0.058	0.063	0.066	0.049	0.060
	1	0.9	0.069	0.070	0.054	0.057	0.051	0.050	0.062	0.052	0.057	0.074	0.057	0.065
		0.95	0.047	0.051	0.038	0.045	0.033	0.037	0.051	0.041	0.046	0.049	0.038	0.050
		1	0.058	0.058	0.041	0.050	0.035	0.046	0.057	0.048	0.049	0.064	0.044	0.058
	10	0.9	0.046	0.059	0.037	0.049	0.040	0.045	0.058	0.045	0.052	0.047	0.039	0.051
		0.95	0.059	0.063	0.049	0.054	0.045	0.051	0.063	0.051	0.054	0.064	0.052	0.067
		1	0.061	0.058	0.049	0.053	0.041	0.048	0.063	0.047	0.053	0.063	0.049	0.065

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 15: Empirical size of panel unit root statistics for 2 known breaks for $N = 40$, $\lambda_1^0 = 0.3$ and $\lambda_2^0 = 0.7$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.049	0.061	0.067	0.052	0.049	0.055	0.064	0.059	0.059	0.058	0.057	0.062
		0.95	0.054	0.054	0.059	0.047	0.042	0.051	0.057	0.054	0.058	0.052	0.054	0.060
		1	0.049	0.055	0.060	0.061	0.061	0.059	0.071	0.064	0.067	0.059	0.055	0.061
	1	0.9	0.049	0.053	0.056	0.049	0.046	0.046	0.062	0.053	0.056	0.058	0.050	0.051
		0.95	0.044	0.051	0.048	0.055	0.051	0.052	0.065	0.061	0.061	0.057	0.055	0.055
		1	0.040	0.051	0.053	0.056	0.047	0.050	0.065	0.061	0.060	0.051	0.049	0.053
	10	0.9	0.046	0.052	0.053	0.058	0.045	0.047	0.062	0.057	0.060	0.053	0.051	0.055
		0.95	0.051	0.052	0.059	0.051	0.045	0.049	0.060	0.052	0.057	0.050	0.052	0.057
		1	0.044	0.046	0.045	0.055	0.051	0.055	0.062	0.058	0.063	0.045	0.047	0.049
100	0.5	0.9	0.062	0.069	0.052	0.041	0.040	0.054	0.046	0.046	0.059	0.044	0.043	0.059
		0.95	0.061	0.074	0.047	0.038	0.036	0.055	0.043	0.045	0.061	0.036	0.042	0.053
		1	0.064	0.077	0.047	0.037	0.039	0.063	0.044	0.050	0.077	0.034	0.040	0.056
	1	0.9	0.054	0.071	0.046	0.042	0.041	0.056	0.044	0.045	0.069	0.033	0.039	0.049
		0.95	0.077	0.084	0.052	0.042	0.044	0.058	0.046	0.050	0.065	0.037	0.043	0.061
		1	0.067	0.087	0.059	0.044	0.049	0.066	0.048	0.053	0.071	0.034	0.046	0.065
	10	0.9	0.054	0.072	0.043	0.029	0.033	0.056	0.037	0.043	0.065	0.024	0.032	0.049
		0.95	0.079	0.091	0.054	0.034	0.034	0.056	0.037	0.041	0.062	0.040	0.047	0.061
		1	0.075	0.091	0.068	0.044	0.044	0.070	0.047	0.054	0.078	0.041	0.057	0.074
200	0.5	0.9	0.060	0.065	0.050	0.072	0.045	0.046	0.084	0.057	0.049	0.067	0.063	0.045
		0.95	0.052	0.066	0.051	0.070	0.056	0.046	0.078	0.064	0.055	0.066	0.066	0.043
		1	0.043	0.048	0.038	0.052	0.037	0.033	0.065	0.049	0.039	0.046	0.050	0.034
	1	0.9	0.047	0.058	0.045	0.058	0.045	0.041	0.064	0.048	0.048	0.055	0.054	0.043
		0.95	0.053	0.057	0.046	0.062	0.052	0.044	0.073	0.054	0.056	0.058	0.060	0.044
		1	0.054	0.054	0.044	0.064	0.046	0.048	0.078	0.051	0.055	0.056	0.057	0.038
	10	0.9	0.049	0.060	0.045	0.065	0.051	0.049	0.070	0.056	0.053	0.056	0.057	0.042
		0.95	0.059	0.055	0.042	0.048	0.038	0.035	0.060	0.046	0.041	0.059	0.056	0.040
		1	0.057	0.059	0.047	0.061	0.053	0.050	0.069	0.055	0.058	0.057	0.061	0.044

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (16) through (19).

Table 16: Empirical size of panel unit root statistics for 2 known breaks for $N = 60$, $\lambda_1^0 = 0.3$ and $\lambda_2^0 = 0.7$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.061	0.071	0.079	0.061	0.054	0.062	0.071	0.067	0.068	0.079	0.071	0.079
		0.95	0.043	0.050	0.058	0.055	0.053	0.056	0.065	0.061	0.067	0.054	0.049	0.055
		1	0.056	0.065	0.072	0.064	0.055	0.066	0.072	0.064	0.072	0.067	0.061	0.072
	1	0.9	0.043	0.042	0.047	0.054	0.054	0.054	0.067	0.057	0.062	0.046	0.043	0.046
		0.95	0.047	0.051	0.056	0.047	0.038	0.046	0.055	0.046	0.053	0.055	0.047	0.053
		1	0.053	0.064	0.061	0.063	0.052	0.058	0.068	0.060	0.065	0.066	0.065	0.063
	10	0.9	0.050	0.046	0.051	0.053	0.045	0.046	0.060	0.050	0.055	0.048	0.043	0.049
		0.95	0.049	0.062	0.065	0.066	0.056	0.056	0.074	0.063	0.070	0.065	0.061	0.064
		1	0.050	0.056	0.064	0.058	0.057	0.060	0.066	0.062	0.072	0.061	0.058	0.065
100	0.5	0.9	0.064	0.079	0.044	0.026	0.032	0.050	0.030	0.035	0.055	0.028	0.032	0.050
		0.95	0.068	0.077	0.045	0.033	0.031	0.060	0.037	0.041	0.066	0.031	0.039	0.053
		1	0.080	0.104	0.061	0.036	0.035	0.066	0.040	0.040	0.077	0.035	0.047	0.069
	1	0.9	0.072	0.083	0.047	0.034	0.039	0.057	0.039	0.044	0.061	0.026	0.033	0.055
		0.95	0.070	0.081	0.049	0.034	0.036	0.052	0.036	0.039	0.058	0.034	0.043	0.055
		1	0.068	0.075	0.042	0.027	0.032	0.046	0.030	0.038	0.057	0.022	0.027	0.046
	10	0.9	0.060	0.077	0.043	0.025	0.033	0.058	0.031	0.039	0.066	0.024	0.036	0.050
		0.95	0.063	0.080	0.037	0.033	0.036	0.056	0.040	0.040	0.066	0.028	0.034	0.045
		1	0.085	0.096	0.065	0.038	0.046	0.071	0.047	0.055	0.082	0.046	0.055	0.071
200	0.5	0.9	0.059	0.056	0.047	0.062	0.040	0.039	0.068	0.047	0.045	0.060	0.058	0.041
		0.95	0.058	0.055	0.040	0.057	0.042	0.038	0.067	0.049	0.044	0.060	0.061	0.035
		1	0.060	0.065	0.046	0.081	0.054	0.051	0.089	0.063	0.056	0.069	0.064	0.042
	1	0.9	0.051	0.061	0.044	0.078	0.057	0.051	0.088	0.062	0.058	0.061	0.061	0.038
		0.95	0.038	0.043	0.026	0.056	0.042	0.039	0.063	0.050	0.049	0.042	0.039	0.021
		1	0.061	0.068	0.051	0.068	0.051	0.050	0.076	0.061	0.056	0.066	0.067	0.044
	10	0.9	0.047	0.055	0.040	0.066	0.047	0.043	0.069	0.056	0.050	0.051	0.056	0.031
		0.95	0.047	0.054	0.038	0.060	0.043	0.038	0.066	0.048	0.043	0.055	0.051	0.033
		1	0.059	0.062	0.050	0.073	0.056	0.050	0.075	0.062	0.055	0.060	0.061	0.045

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (16) through (19).

Table 17: Empirical size of panel unit root statistics for 1 unknown break for $N = 40$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.050	0.078	0.061	0.054	0.073	0.054	0.057	0.080	0.061	0.057	0.071	0.063
		0.95	0.049	0.084	0.068	0.045	0.065	0.052	0.052	0.076	0.062	0.053	0.082	0.068
		1	0.059	0.090	0.079	0.052	0.073	0.057	0.060	0.082	0.067	0.069	0.094	0.082
	1	0.9	0.053	0.101	0.081	0.072	0.099	0.090	0.080	0.114	0.097	0.068	0.094	0.079
		0.95	0.067	0.099	0.088	0.078	0.093	0.085	0.079	0.099	0.091	0.073	0.096	0.089
		1	0.080	0.121	0.103	0.073	0.104	0.089	0.085	0.115	0.101	0.091	0.117	0.101
	10	0.9	0.160	0.224	0.215	0.195	0.226	0.220	0.202	0.232	0.225	0.193	0.221	0.218
		0.95	0.182	0.239	0.231	0.220	0.250	0.245	0.226	0.257	0.255	0.205	0.235	0.234
		1	0.156	0.215	0.201	0.200	0.232	0.219	0.206	0.239	0.231	0.188	0.201	0.200
100	0.5	0.9	0.066	0.055	0.076	0.037	0.037	0.048	0.042	0.040	0.057	0.060	0.060	0.077
		0.95	0.069	0.059	0.083	0.039	0.040	0.048	0.043	0.044	0.057	0.065	0.062	0.082
		1	0.054	0.050	0.087	0.034	0.033	0.047	0.039	0.040	0.053	0.052	0.055	0.086
	1	0.9	0.064	0.057	0.091	0.035	0.036	0.051	0.043	0.048	0.067	0.057	0.064	0.092
		0.95	0.050	0.043	0.064	0.033	0.035	0.043	0.040	0.038	0.048	0.045	0.047	0.065
		1	0.072	0.068	0.099	0.055	0.052	0.067	0.060	0.060	0.072	0.073	0.070	0.096
	10	0.9	0.132	0.135	0.160	0.106	0.113	0.133	0.109	0.124	0.138	0.125	0.139	0.160
		0.95	0.144	0.167	0.191	0.143	0.154	0.169	0.148	0.160	0.173	0.154	0.169	0.192
		1	0.137	0.160	0.173	0.148	0.151	0.159	0.150	0.154	0.163	0.146	0.155	0.172
200	0.5	0.9	0.077	0.040	0.055	0.023	0.020	0.021	0.027	0.025	0.025	0.053	0.050	0.043
		0.95	0.088	0.043	0.055	0.026	0.025	0.024	0.029	0.026	0.027	0.053	0.047	0.043
		1	0.085	0.047	0.069	0.029	0.030	0.029	0.035	0.036	0.033	0.053	0.055	0.059
	1	0.9	0.088	0.049	0.063	0.024	0.025	0.023	0.030	0.032	0.031	0.057	0.061	0.057
		0.95	0.110	0.060	0.078	0.034	0.033	0.035	0.037	0.040	0.039	0.071	0.073	0.070
		1	0.108	0.060	0.079	0.038	0.038	0.036	0.044	0.044	0.045	0.072	0.072	0.073
	10	0.9	0.111	0.072	0.098	0.044	0.046	0.049	0.046	0.049	0.051	0.080	0.087	0.086
		0.95	0.124	0.065	0.101	0.042	0.043	0.045	0.044	0.050	0.050	0.088	0.091	0.087
		1	0.113	0.076	0.098	0.052	0.051	0.057	0.055	0.060	0.061	0.084	0.089	0.086

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 18: Empirical size of panel unit root statistics for 1 unknown break for $N = 60$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.058	0.090	0.067	0.047	0.081	0.061	0.055	0.087	0.065	0.059	0.087	0.065
		0.95	0.063	0.107	0.092	0.060	0.093	0.077	0.066	0.103	0.081	0.069	0.101	0.090
		1	0.070	0.111	0.088	0.065	0.096	0.079	0.074	0.103	0.091	0.067	0.104	0.086
	1	0.9	0.069	0.136	0.119	0.078	0.120	0.102	0.081	0.133	0.112	0.084	0.130	0.118
		0.95	0.063	0.125	0.102	0.081	0.107	0.090	0.087	0.115	0.097	0.075	0.117	0.101
		1	0.080	0.133	0.115	0.088	0.123	0.101	0.095	0.128	0.110	0.093	0.126	0.111
	10	0.9	0.168	0.221	0.210	0.205	0.230	0.227	0.207	0.232	0.229	0.190	0.218	0.217
		0.95	0.193	0.250	0.235	0.219	0.251	0.256	0.222	0.258	0.257	0.215	0.238	0.239
		1	0.198	0.256	0.248	0.236	0.264	0.256	0.239	0.264	0.260	0.214	0.247	0.252
100	0.5	0.9	0.057	0.047	0.076	0.035	0.037	0.047	0.035	0.038	0.049	0.052	0.055	0.074
		0.95	0.072	0.044	0.072	0.035	0.032	0.043	0.040	0.037	0.051	0.057	0.056	0.075
		1	0.072	0.051	0.091	0.038	0.036	0.056	0.041	0.041	0.062	0.062	0.061	0.091
	1	0.9	0.054	0.052	0.080	0.039	0.039	0.054	0.041	0.044	0.059	0.051	0.056	0.080
		0.95	0.066	0.061	0.087	0.040	0.045	0.059	0.045	0.051	0.062	0.059	0.068	0.090
		1	0.079	0.067	0.109	0.050	0.052	0.068	0.053	0.055	0.070	0.073	0.080	0.106
	10	0.9	0.103	0.109	0.148	0.081	0.084	0.111	0.088	0.090	0.116	0.102	0.111	0.149
		0.95	0.174	0.186	0.223	0.160	0.167	0.186	0.164	0.167	0.193	0.178	0.191	0.223
		1	0.153	0.172	0.194	0.148	0.155	0.175	0.154	0.161	0.180	0.166	0.173	0.194
200	0.5	0.9	0.117	0.060	0.086	0.037	0.037	0.036	0.041	0.039	0.041	0.069	0.076	0.072
		0.95	0.135	0.057	0.091	0.035	0.036	0.037	0.037	0.039	0.042	0.075	0.081	0.076
		1	0.138	0.053	0.081	0.026	0.026	0.027	0.031	0.030	0.031	0.067	0.071	0.071
	1	0.9	0.127	0.058	0.091	0.029	0.030	0.030	0.033	0.037	0.034	0.070	0.078	0.077
		0.95	0.143	0.059	0.098	0.035	0.036	0.036	0.036	0.038	0.040	0.082	0.084	0.078
		1	0.150	0.053	0.084	0.027	0.027	0.027	0.033	0.031	0.029	0.072	0.078	0.072
	10	0.9	0.162	0.088	0.123	0.043	0.046	0.046	0.047	0.047	0.053	0.107	0.110	0.110
		0.95	0.161	0.080	0.123	0.051	0.054	0.055	0.053	0.058	0.061	0.099	0.107	0.109
		1	0.178	0.110	0.142	0.075	0.083	0.085	0.083	0.090	0.089	0.123	0.124	0.126

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 19: Empirical size of panel unit root statistics for 2 unknown breaks for $N = 40$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.033	0.043	0.045	0.046	0.043	0.042	0.051	0.050	0.047	0.049	0.045	0.045
		0.95	0.029	0.039	0.038	0.037	0.034	0.038	0.039	0.036	0.044	0.035	0.036	0.043
		1	0.028	0.039	0.038	0.037	0.034	0.039	0.041	0.040	0.044	0.036	0.039	0.045
	1	0.9	0.035	0.042	0.038	0.040	0.038	0.041	0.043	0.042	0.045	0.043	0.041	0.044
		0.95	0.024	0.031	0.035	0.031	0.031	0.037	0.035	0.037	0.040	0.031	0.031	0.036
		1	0.020	0.034	0.036	0.031	0.031	0.030	0.034	0.034	0.034	0.032	0.034	0.038
	10	0.9	0.096	0.120	0.119	0.127	0.132	0.136	0.131	0.135	0.138	0.117	0.120	0.128
		0.95	0.132	0.164	0.158	0.165	0.167	0.180	0.170	0.174	0.187	0.160	0.163	0.165
		1	0.110	0.132	0.131	0.132	0.134	0.144	0.136	0.140	0.146	0.127	0.133	0.138
100	0.5	0.9	0.073	0.058	0.075	0.041	0.035	0.041	0.047	0.037	0.045	0.073	0.060	0.077
		0.95	0.052	0.044	0.063	0.033	0.027	0.039	0.038	0.032	0.044	0.053	0.050	0.060
		1	0.058	0.051	0.071	0.042	0.030	0.040	0.043	0.034	0.044	0.056	0.055	0.071
	1	0.9	0.060	0.050	0.069	0.043	0.034	0.046	0.052	0.038	0.049	0.057	0.053	0.070
		0.95	0.040	0.037	0.050	0.035	0.024	0.029	0.039	0.030	0.034	0.047	0.041	0.049
		1	0.054	0.046	0.061	0.042	0.036	0.047	0.054	0.041	0.052	0.052	0.051	0.061
	10	0.9	0.071	0.079	0.092	0.072	0.065	0.074	0.076	0.068	0.077	0.076	0.074	0.092
		0.95	0.080	0.089	0.102	0.092	0.084	0.091	0.094	0.088	0.091	0.089	0.093	0.104
		1	0.092	0.089	0.103	0.090	0.084	0.091	0.095	0.087	0.094	0.092	0.091	0.105
200	0.5	0.9	0.159	0.049	0.090	0.026	0.024	0.022	0.027	0.026	0.025	0.068	0.056	0.053
		0.95	0.195	0.056	0.106	0.029	0.023	0.022	0.032	0.026	0.025	0.075	0.064	0.060
		1	0.206	0.055	0.110	0.026	0.019	0.021	0.029	0.022	0.021	0.082	0.065	0.063
	1	0.9	0.181	0.045	0.090	0.019	0.016	0.016	0.023	0.016	0.018	0.065	0.048	0.050
		0.95	0.206	0.057	0.105	0.027	0.019	0.018	0.029	0.023	0.019	0.081	0.068	0.066
		1	0.210	0.056	0.096	0.032	0.027	0.027	0.034	0.030	0.032	0.078	0.068	0.064
	10	0.9	0.187	0.068	0.108	0.042	0.038	0.040	0.042	0.043	0.040	0.085	0.076	0.075
		0.95	0.193	0.066	0.100	0.036	0.028	0.028	0.039	0.030	0.032	0.080	0.069	0.068
		1	0.147	0.052	0.091	0.036	0.031	0.032	0.041	0.035	0.037	0.073	0.062	0.061

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (16) through (19).

Table 20: Empirical size of panel unit root statistics for 2 unknown breaks for $N = 60$

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.036	0.041	0.041	0.033	0.028	0.038	0.040	0.038	0.038	0.039	0.038	0.041
		0.95	0.030	0.041	0.038	0.047	0.042	0.046	0.049	0.047	0.051	0.037	0.039	0.038
		1	0.034	0.039	0.041	0.041	0.040	0.044	0.042	0.041	0.044	0.037	0.039	0.042
	1	0.9	0.032	0.040	0.039	0.036	0.031	0.038	0.036	0.035	0.042	0.036	0.039	0.041
		0.95	0.042	0.054	0.056	0.045	0.046	0.051	0.048	0.053	0.053	0.051	0.056	0.058
		1	0.035	0.048	0.046	0.042	0.043	0.047	0.044	0.046	0.051	0.047	0.047	0.047
	10	0.9	0.117	0.136	0.136	0.145	0.153	0.161	0.146	0.156	0.162	0.133	0.139	0.143
		0.95	0.131	0.156	0.152	0.157	0.163	0.170	0.160	0.167	0.172	0.153	0.159	0.164
		1	0.121	0.149	0.148	0.148	0.155	0.158	0.153	0.156	0.164	0.144	0.151	0.152
100	0.5	0.9	0.079	0.067	0.097	0.060	0.042	0.057	0.075	0.048	0.063	0.085	0.076	0.096
		0.95	0.087	0.072	0.100	0.056	0.044	0.058	0.066	0.049	0.064	0.084	0.080	0.098
		1	0.091	0.075	0.110	0.065	0.052	0.065	0.066	0.058	0.068	0.091	0.086	0.109
	1	0.9	0.057	0.040	0.069	0.036	0.029	0.038	0.042	0.031	0.043	0.055	0.053	0.069
		0.95	0.062	0.054	0.074	0.046	0.034	0.042	0.048	0.040	0.046	0.067	0.058	0.074
		1	0.062	0.046	0.070	0.036	0.031	0.036	0.037	0.032	0.037	0.053	0.048	0.069
	10	0.9	0.096	0.100	0.105	0.096	0.087	0.100	0.102	0.090	0.102	0.102	0.100	0.106
		0.95	0.082	0.092	0.105	0.097	0.091	0.095	0.098	0.093	0.101	0.090	0.092	0.104
		1	0.094	0.096	0.101	0.101	0.095	0.101	0.107	0.097	0.102	0.097	0.091	0.102
200	0.5	0.9	0.230	0.059	0.133	0.031	0.025	0.025	0.032	0.027	0.028	0.096	0.073	0.070
		0.95	0.252	0.057	0.121	0.028	0.023	0.022	0.031	0.025	0.024	0.084	0.068	0.063
		1	0.301	0.086	0.169	0.042	0.034	0.033	0.045	0.037	0.036	0.114	0.098	0.090
	1	0.9	0.251	0.060	0.122	0.032	0.025	0.025	0.035	0.026	0.026	0.089	0.075	0.070
		0.95	0.279	0.070	0.139	0.033	0.029	0.031	0.036	0.032	0.032	0.100	0.083	0.078
		1	0.302	0.075	0.150	0.034	0.029	0.028	0.036	0.030	0.033	0.108	0.090	0.087
	10	0.9	0.268	0.090	0.146	0.042	0.037	0.041	0.047	0.040	0.045	0.111	0.102	0.098
		0.95	0.226	0.075	0.126	0.052	0.044	0.045	0.055	0.049	0.048	0.087	0.083	0.081
		1	0.213	0.064	0.116	0.045	0.039	0.039	0.045	0.041	0.040	0.084	0.071	0.071

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (16) through (19).

Table 21: Empirical size of panel unit root statistics for 1 known break for $N = 20$ and $\lambda^0 = 0.5$ using iterations

T	σ_F^2	ρ	MSB_m MSB^{GLS}	MSB_m MZ_α^{GLS}	MSB_m MZ_t^{GLS}	P MSB^{GLS}	P MZ_α^{GLS}	P MZ_t^{GLS}	P_m MSB^{GLS}	P_m MZ_α^{GLS}	P_m MZ_t^{GLS}	Z MSB^{GLS}	Z MZ_α^{GLS}	Z MZ_t^{GLS}
50	0.5	0.9	0.044	0.053	0.049	0.044	0.050	0.049	0.055	0.063	0.059	0.047	0.049	0.052
		0.95	0.048	0.061	0.065	0.055	0.064	0.059	0.069	0.070	0.066	0.057	0.060	0.061
		1	0.052	0.063	0.058	0.050	0.057	0.059	0.063	0.076	0.075	0.061	0.061	0.061
	1	0.9	0.032	0.038	0.034	0.045	0.045	0.044	0.054	0.060	0.053	0.032	0.036	0.038
		0.95	0.052	0.069	0.065	0.055	0.056	0.052	0.071	0.069	0.065	0.066	0.072	0.065
		1	0.046	0.054	0.050	0.036	0.044	0.047	0.050	0.056	0.059	0.045	0.048	0.049
	10	0.9	0.049	0.061	0.052	0.057	0.051	0.046	0.066	0.062	0.057	0.059	0.060	0.055
		0.95	0.043	0.078	0.064	0.060	0.067	0.061	0.074	0.079	0.076	0.063	0.069	0.067
		1	0.041	0.060	0.055	0.065	0.066	0.059	0.075	0.074	0.073	0.054	0.054	0.057
100	0.5	0.9	0.038	0.049	0.048	0.058	0.058	0.050	0.072	0.077	0.065	0.048	0.054	0.046
		0.95	0.047	0.065	0.066	0.060	0.065	0.062	0.078	0.080	0.066	0.066	0.071	0.060
		1	0.051	0.055	0.061	0.049	0.055	0.044	0.061	0.063	0.051	0.059	0.068	0.054
	1	0.9	0.044	0.051	0.056	0.052	0.055	0.048	0.061	0.063	0.059	0.052	0.058	0.055
		0.95	0.037	0.043	0.049	0.046	0.049	0.045	0.057	0.058	0.051	0.052	0.057	0.047
		1	0.038	0.048	0.049	0.047	0.053	0.047	0.059	0.059	0.052	0.052	0.058	0.043
	10	0.9	0.046	0.055	0.057	0.063	0.063	0.054	0.072	0.076	0.066	0.058	0.064	0.053
		0.95	0.041	0.049	0.048	0.055	0.056	0.050	0.069	0.071	0.059	0.056	0.056	0.048
		1	0.041	0.059	0.066	0.062	0.062	0.052	0.073	0.076	0.063	0.063	0.070	0.060
200	0.5	0.9	0.063	0.069	0.053	0.059	0.050	0.049	0.070	0.063	0.065	0.064	0.058	0.060
		0.95	0.048	0.058	0.048	0.046	0.040	0.041	0.055	0.052	0.051	0.051	0.045	0.050
		1	0.062	0.075	0.059	0.058	0.056	0.054	0.070	0.063	0.068	0.074	0.061	0.068
	1	0.9	0.065	0.066	0.053	0.053	0.045	0.044	0.067	0.052	0.060	0.066	0.053	0.056
		0.95	0.051	0.060	0.050	0.046	0.041	0.042	0.057	0.052	0.054	0.058	0.049	0.062
		1	0.059	0.065	0.054	0.057	0.050	0.046	0.068	0.064	0.060	0.061	0.056	0.053
	10	0.9	0.060	0.071	0.059	0.061	0.057	0.059	0.072	0.065	0.064	0.067	0.059	0.064
		0.95	0.050	0.058	0.047	0.052	0.049	0.054	0.068	0.060	0.060	0.053	0.045	0.051
		1	0.051	0.069	0.054	0.063	0.057	0.060	0.071	0.066	0.064	0.060	0.053	0.061

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (12) through (15).

Table 22: Empirical size of panel unit root statistics for 2 known breaks for N=20 and $\lambda^0 = 0.5$ using iterations

			MSB_m			P			P_m			Z		
T	σ_F^2	ρ	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MZ_α^{GLS}	MZ_t^{GLS}
50	0.5	0.9	0.043	0.050	0.046	0.047	0.048	0.048	0.062	0.060	0.061	0.050	0.047	0.050
		0.95	0.054	0.058	0.067	0.062	0.056	0.055	0.070	0.064	0.066	0.063	0.062	0.070
		1	0.042	0.055	0.056	0.054	0.047	0.042	0.067	0.057	0.059	0.059	0.053	0.057
	1	0.9	0.038	0.057	0.053	0.048	0.048	0.050	0.059	0.055	0.062	0.051	0.054	0.056
		0.95	0.044	0.051	0.052	0.058	0.057	0.057	0.075	0.069	0.068	0.057	0.049	0.054
		1	0.055	0.061	0.062	0.056	0.055	0.054	0.072	0.067	0.065	0.065	0.060	0.062
	10	0.9	0.039	0.053	0.059	0.052	0.050	0.056	0.062	0.060	0.071	0.051	0.052	0.061
		0.95	0.046	0.059	0.065	0.047	0.047	0.053	0.056	0.058	0.061	0.056	0.058	0.062
		1	0.039	0.048	0.060	0.049	0.047	0.045	0.063	0.060	0.060	0.050	0.050	0.057
100	0.5	0.9	0.050	0.068	0.044	0.038	0.038	0.055	0.048	0.051	0.063	0.038	0.041	0.051
		0.95	0.064	0.078	0.056	0.048	0.050	0.065	0.054	0.061	0.076	0.048	0.052	0.060
		1	0.060	0.077	0.059	0.040	0.045	0.056	0.049	0.056	0.073	0.043	0.044	0.063
	1	0.9	0.073	0.064	0.052	0.034	0.034	0.047	0.041	0.041	0.063	0.039	0.045	0.056
		0.95	0.054	0.069	0.051	0.040	0.044	0.056	0.050	0.056	0.069	0.038	0.042	0.052
		1	0.075	0.082	0.062	0.047	0.051	0.065	0.058	0.064	0.075	0.052	0.057	0.070
	10	0.9	0.075	0.092	0.069	0.056	0.057	0.068	0.064	0.063	0.081	0.055	0.059	0.075
		0.95	0.046	0.059	0.047	0.034	0.034	0.045	0.042	0.044	0.051	0.039	0.042	0.050
		1	0.062	0.082	0.064	0.046	0.047	0.059	0.054	0.054	0.069	0.047	0.051	0.068
200	0.5	0.9	0.048	0.057	0.046	0.064	0.051	0.047	0.075	0.058	0.061	0.061	0.057	0.041
		0.95	0.048	0.052	0.043	0.051	0.043	0.045	0.068	0.053	0.054	0.047	0.047	0.039
		1	0.064	0.065	0.060	0.075	0.064	0.064	0.085	0.072	0.068	0.067	0.062	0.057
	1	0.9	0.046	0.054	0.045	0.062	0.048	0.049	0.073	0.059	0.063	0.050	0.050	0.040
		0.95	0.053	0.057	0.045	0.055	0.047	0.046	0.072	0.053	0.054	0.052	0.048	0.038
		1	0.064	0.065	0.056	0.070	0.057	0.056	0.079	0.066	0.064	0.066	0.064	0.057
	10	0.9	0.060	0.065	0.057	0.060	0.043	0.045	0.077	0.059	0.062	0.063	0.061	0.047
		0.95	0.053	0.065	0.051	0.061	0.047	0.047	0.071	0.058	0.058	0.058	0.056	0.046
		1	0.054	0.063	0.052	0.066	0.050	0.051	0.081	0.066	0.062	0.060	0.057	0.047

Simulations are based on 1000 replications considering the 5% level of significance. The DGP is given by Equations (16) through (19).